

# Precision measurement of the positive muon lifetime by the MuLan collaboration



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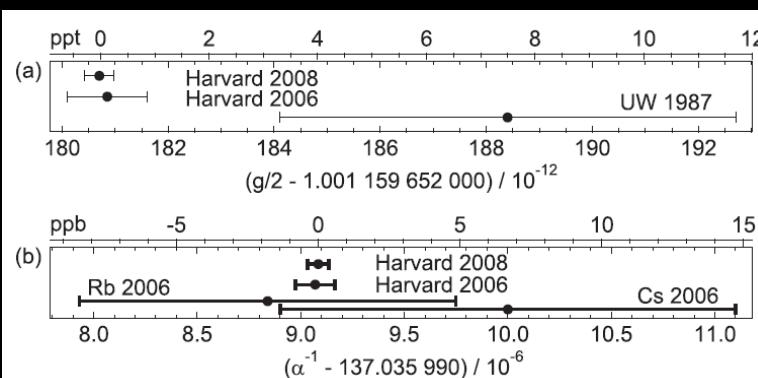
- Why?
- How and where?
- Problems / challenges
- Data analysis
- Consistency checks
- Results

# Standard Model

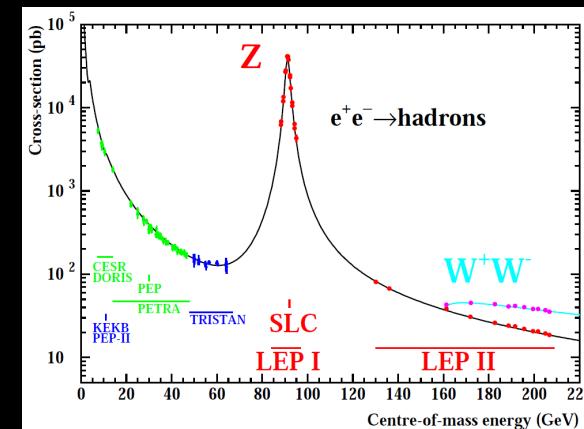
The predictive power of the Standard Model depends  
on well-measured input parameters

$$\alpha^{-1}$$

PDG-2010       $0.00037 \text{ ppm}$   
 $137.035999084(51)$

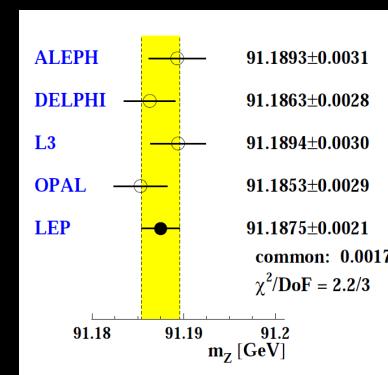


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$$M_Z$$

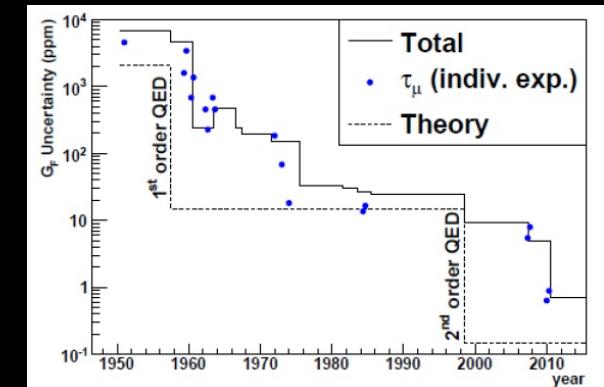
23 ppm  
 $91.1876(21) \text{ GeV}$



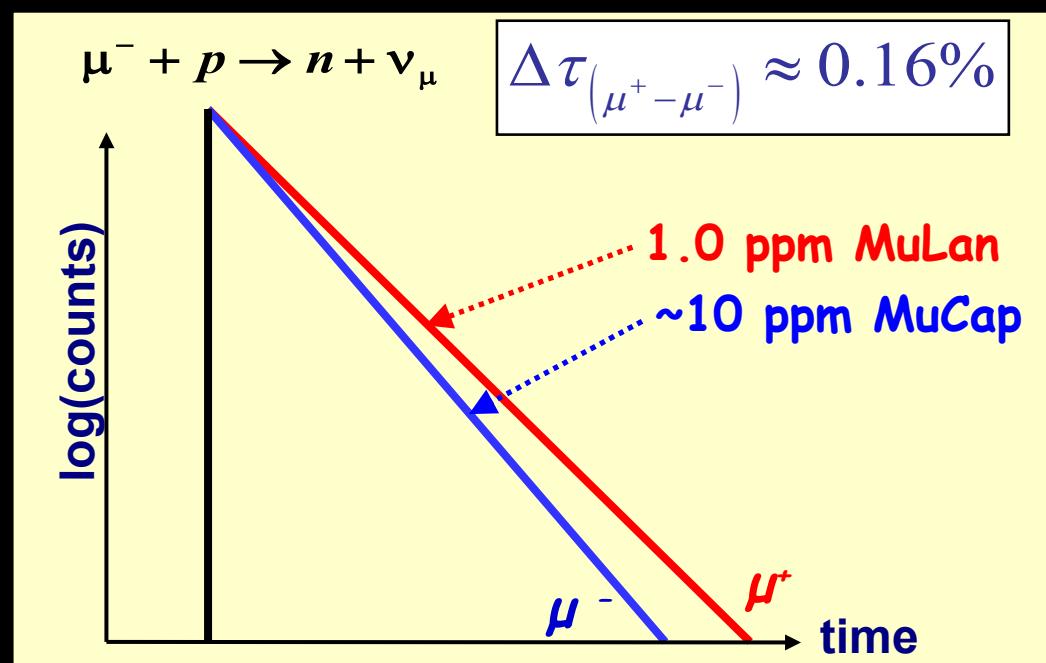
$$G_F$$

8.6 ppm  
 $1.16639(9) \times 10^{-5}^*$   
 $\text{GeV}^{-2}$

from muon lifetime



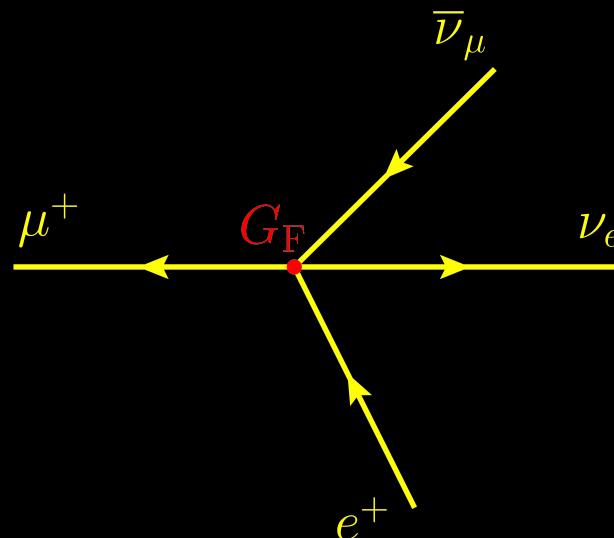
- $\tau_\mu$  can be used for the most precise determination of Fermi constant  $G_F$
  - $\tau_\mu$  is needed for “reference” lifetime for precision muon capture experiments
    - MuCap:  $\mu^- + p$
    - MuSun:  $\mu^- + d$
- capture rate from “lifetime” difference of  $\mu^-$  and  $\mu^+$



The singlet capture rate  $\Lambda_s$  is used to determine  $g_p$  and compare with theory

$$\Lambda_s = \frac{1}{\tau_{\mu^-}^d} - \frac{1}{\tau_{\mu^+}}$$

# Determination of $G_F$ from $\tau_\mu$



$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + \Delta q)$$

QED, QCD rad. corrections

$$\frac{\delta G_F}{G_F} = \frac{1}{2} \sqrt{\left( \frac{\delta \tau_\mu}{\tau_\mu} \right)^2 + \left( 5 \frac{\delta m_\mu}{m_\mu} \right)^2 + \left( \frac{\delta(\Delta q)}{1 + \Delta q} \right)^2}$$



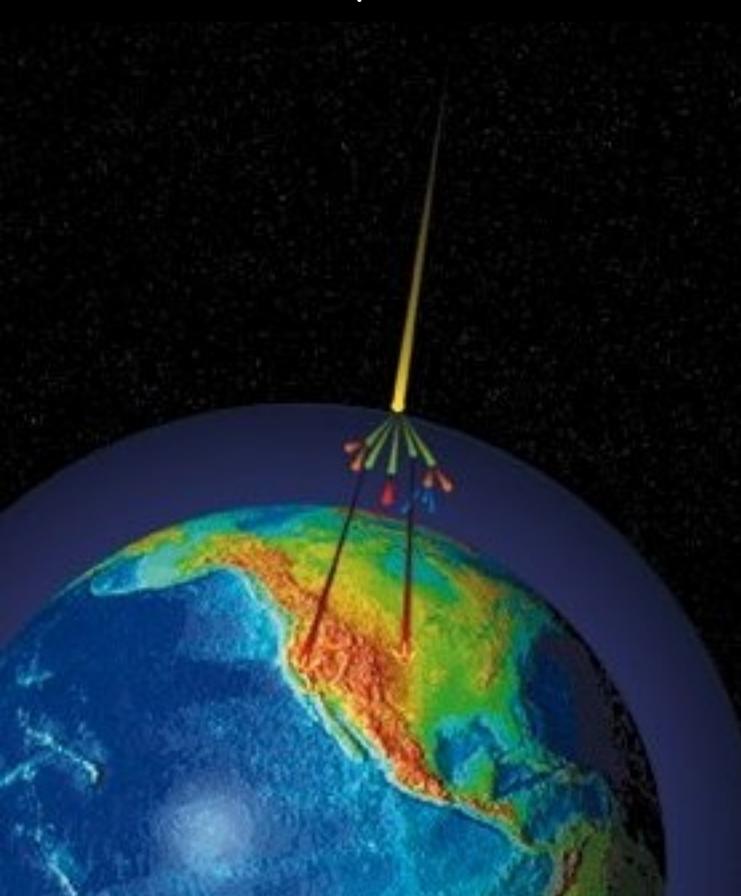
# Where to find muons?

$$m_\mu = 105.7 \text{ MeV}$$

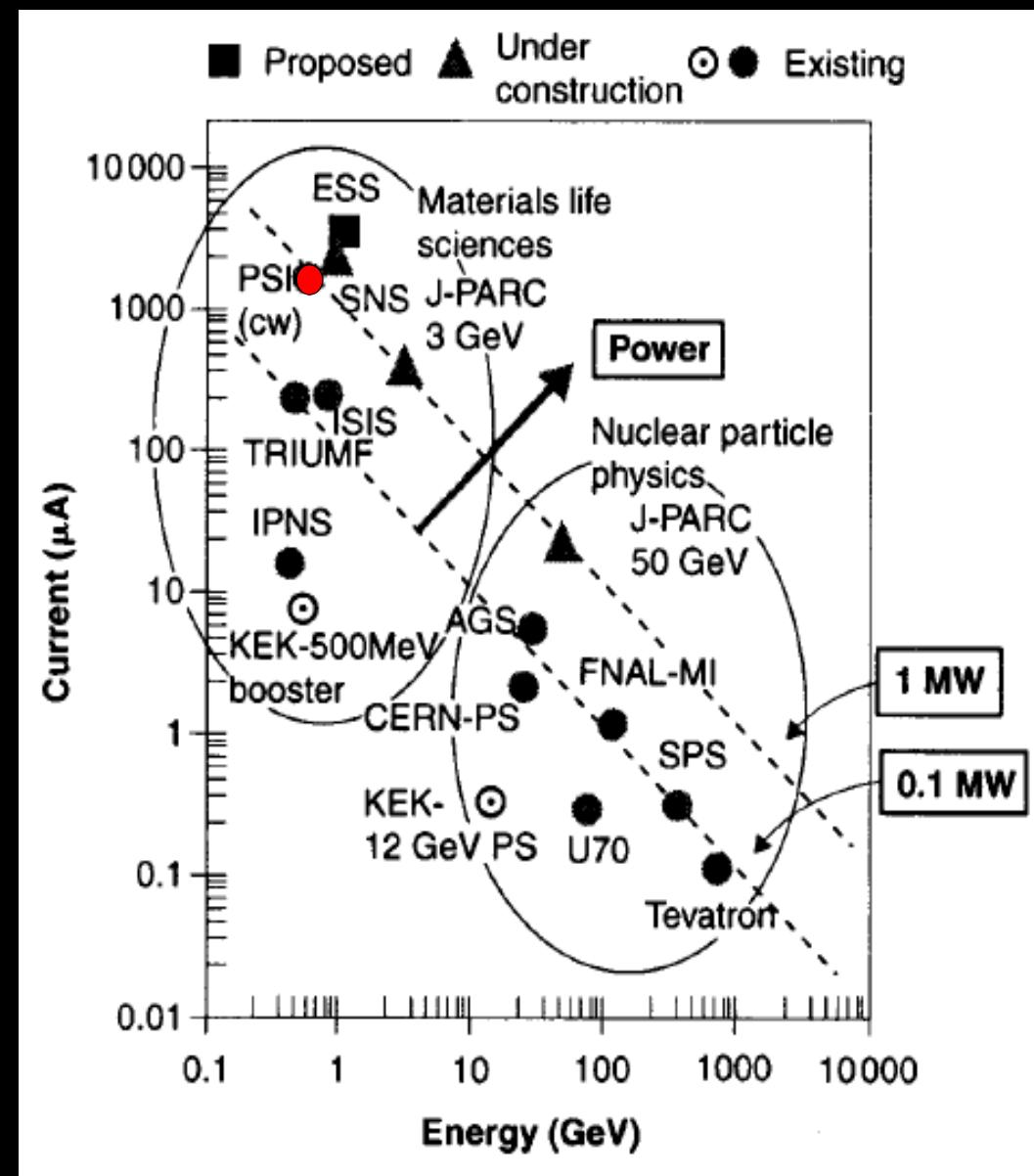
Cosmic rays:

- $\sim 1 \text{ muon}/(\text{cm}^2 \text{ min})$  at sea level
- $\langle E \rangle \sim 4 \text{ GeV}$  at sea level

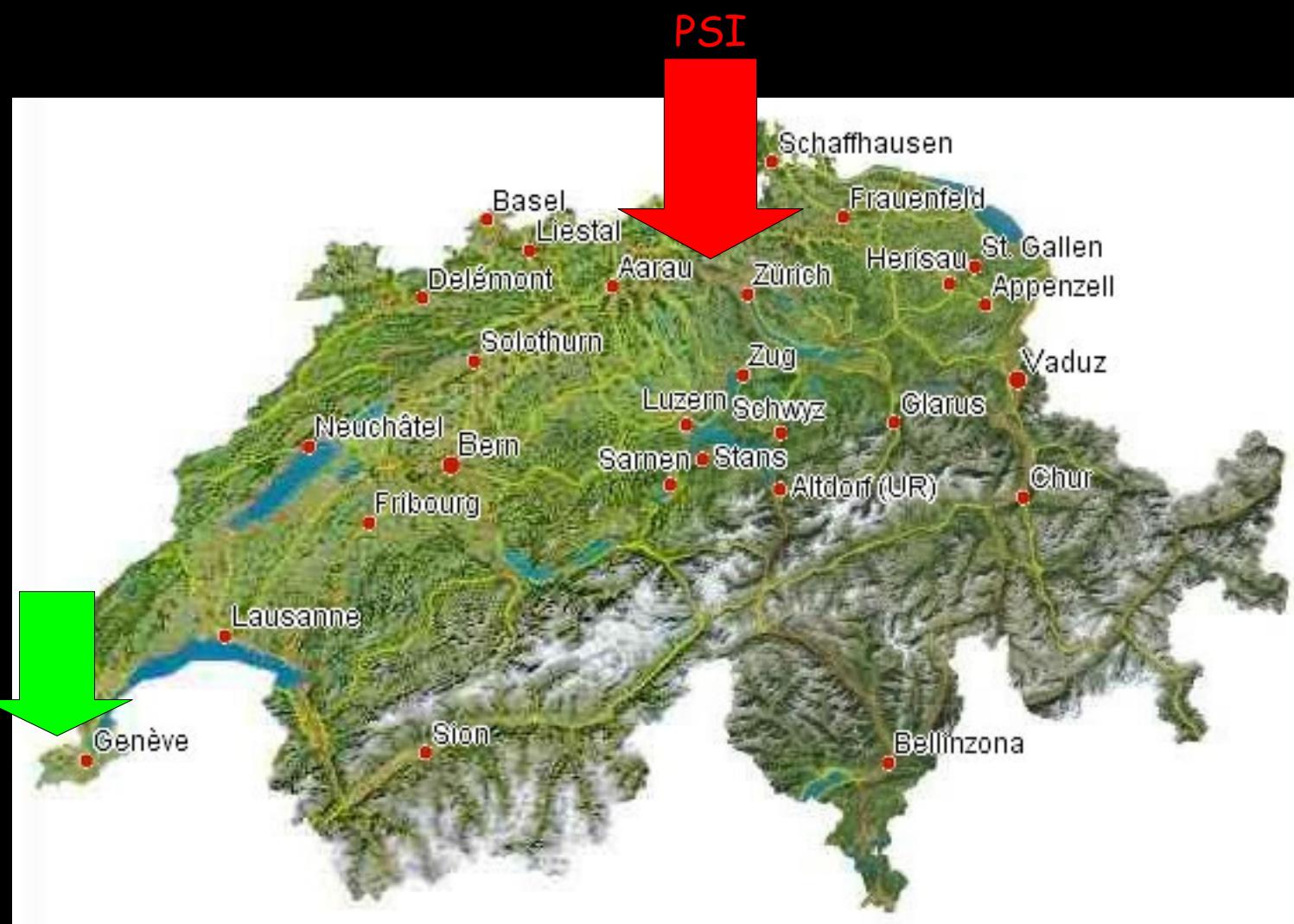
1 ppm  $\rightarrow 10^{12}$  muon decays  
 $\rightarrow$  200 years for  $10 \times 10 \text{ m}^2$  detector



Muon beams worldwide (2007)

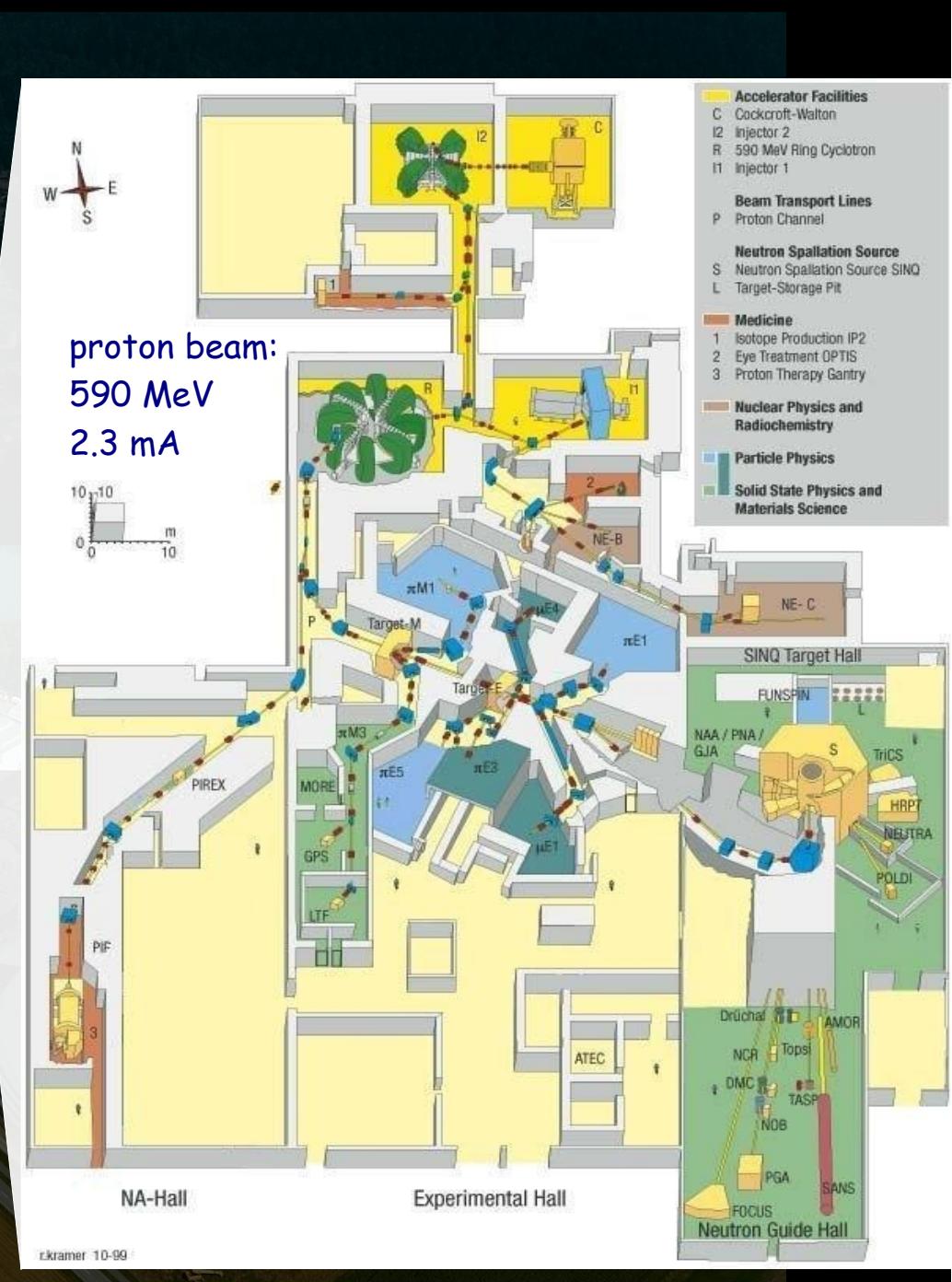


# We go to Switzerland to find muons

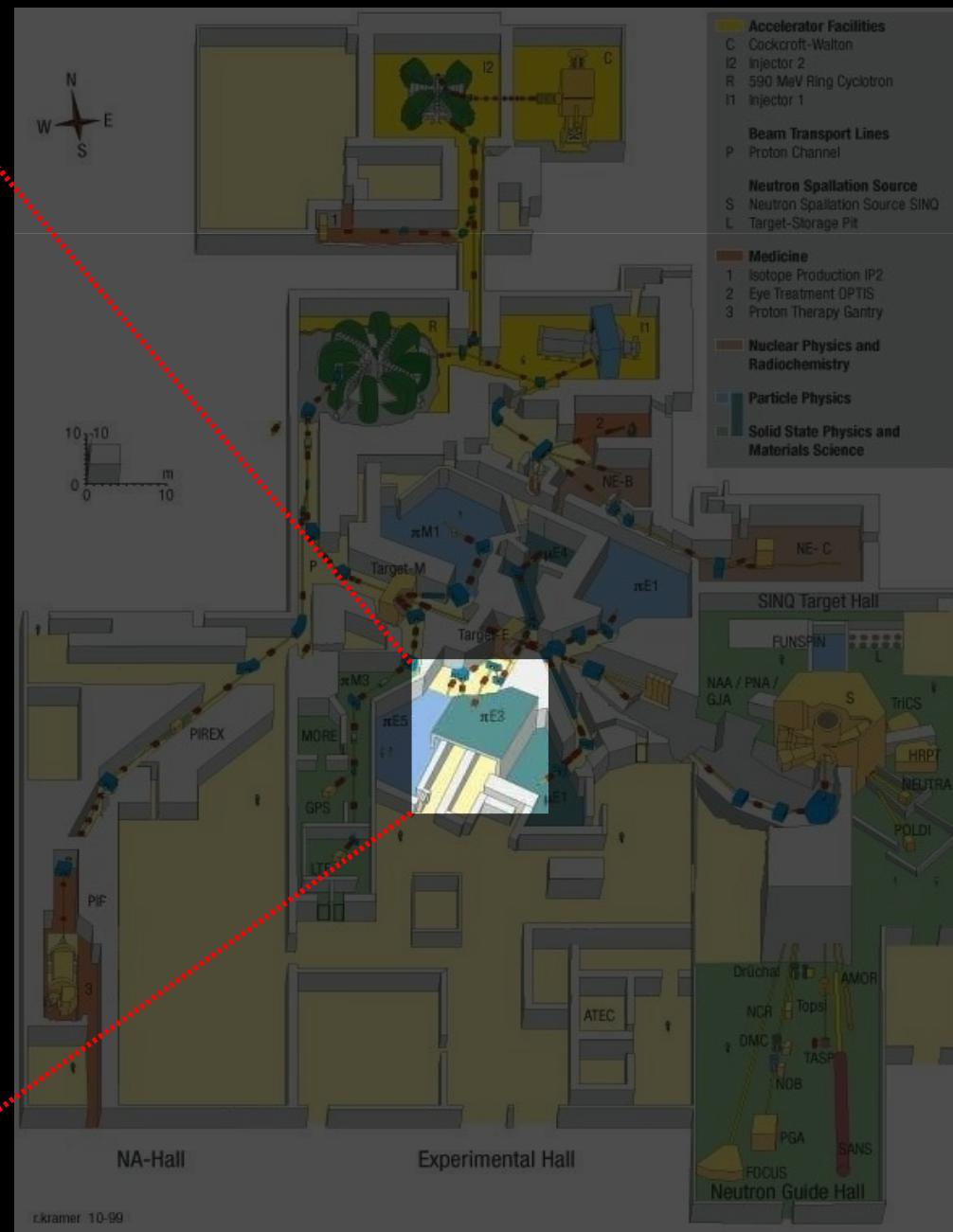
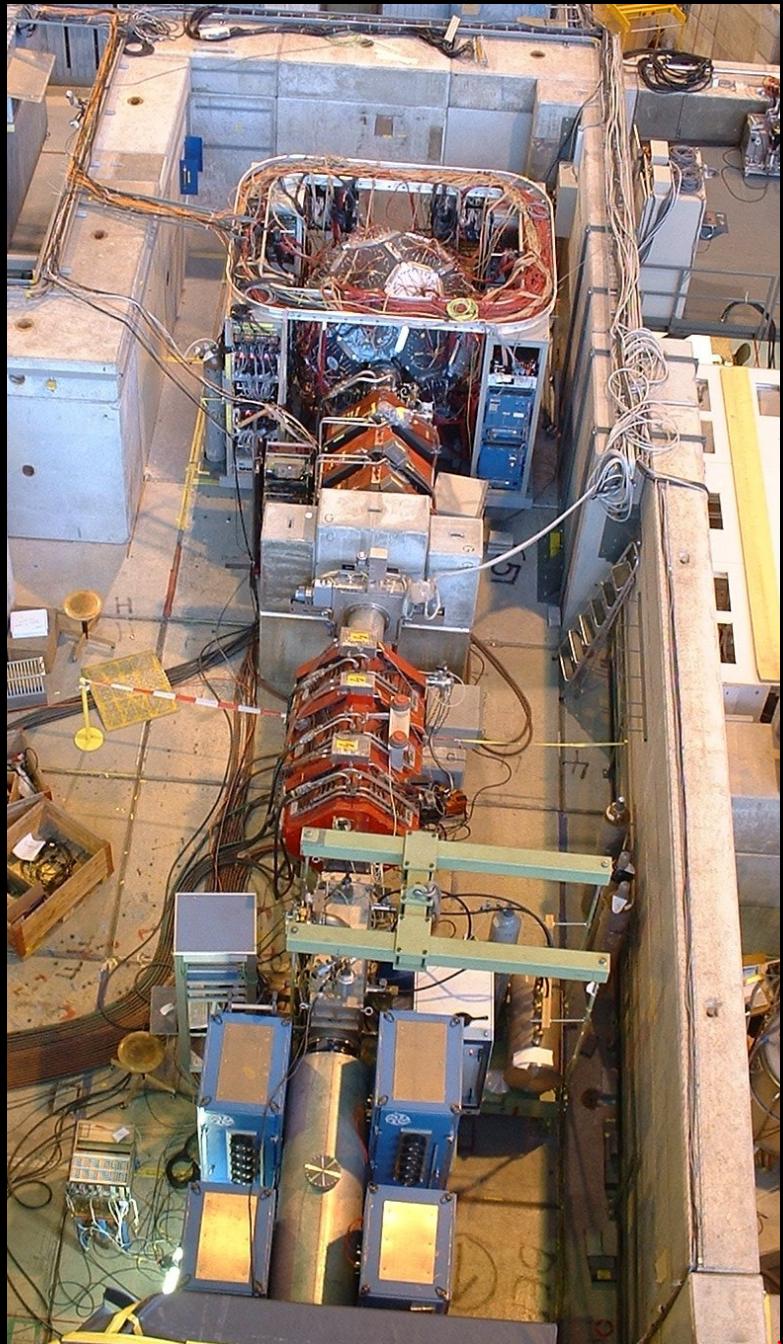


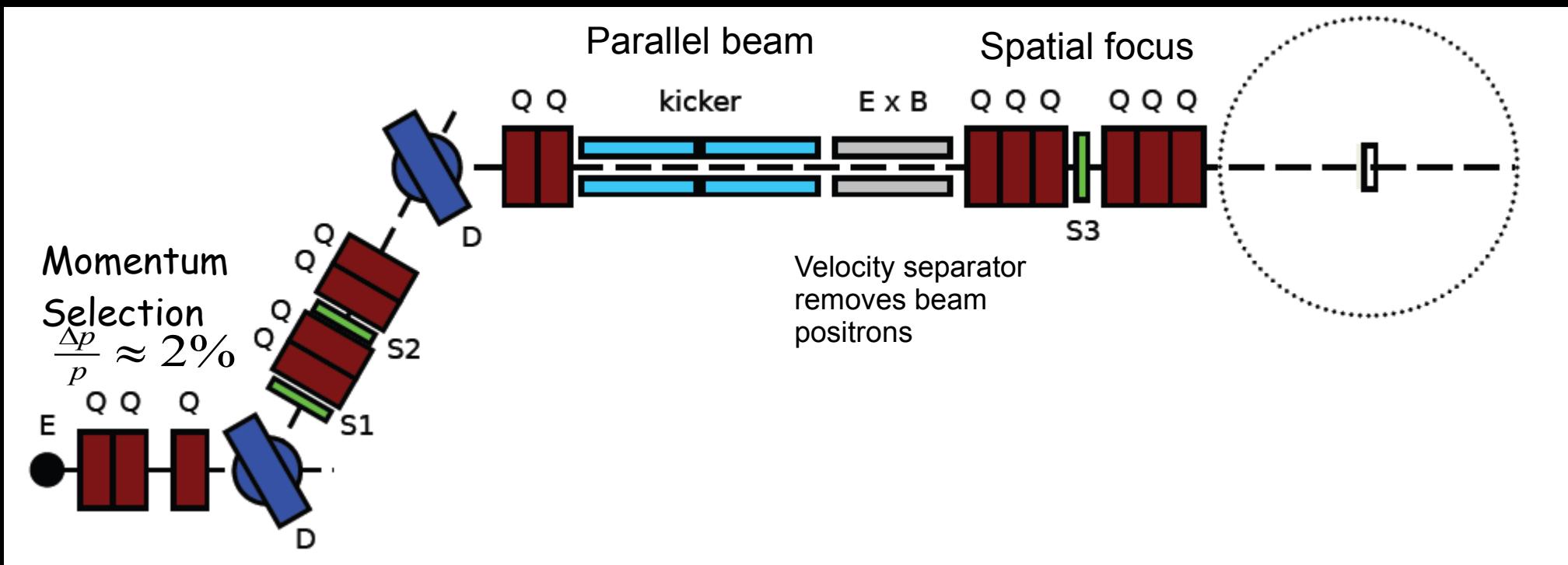


# $\pi E3$ Beamline at PSI



# $\pi$ E3 Beamline at PSI





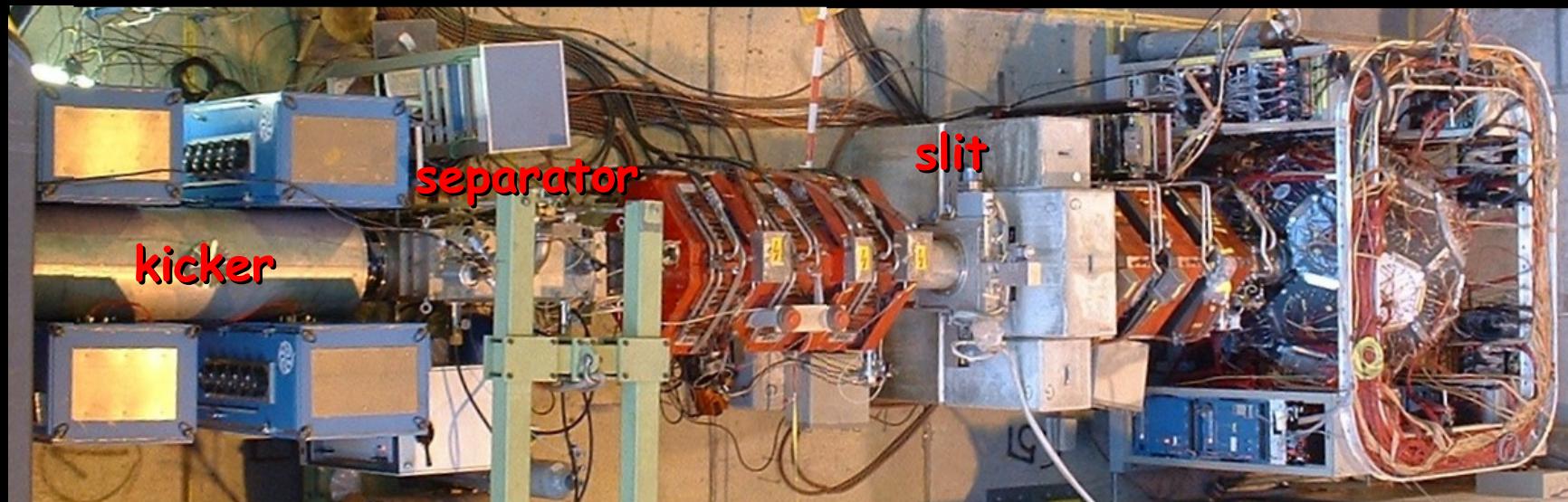
# The Kicker is used to create a pulsed beam

beam

kicker

separator

slit



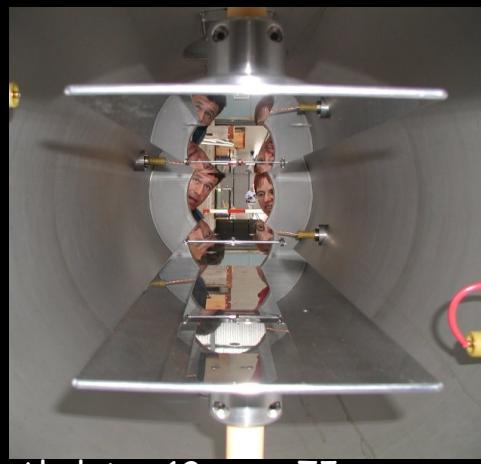
kicker

beam

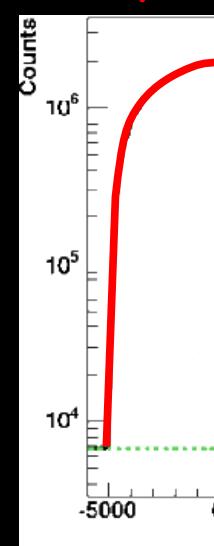
slit

target

5 $\mu$ s

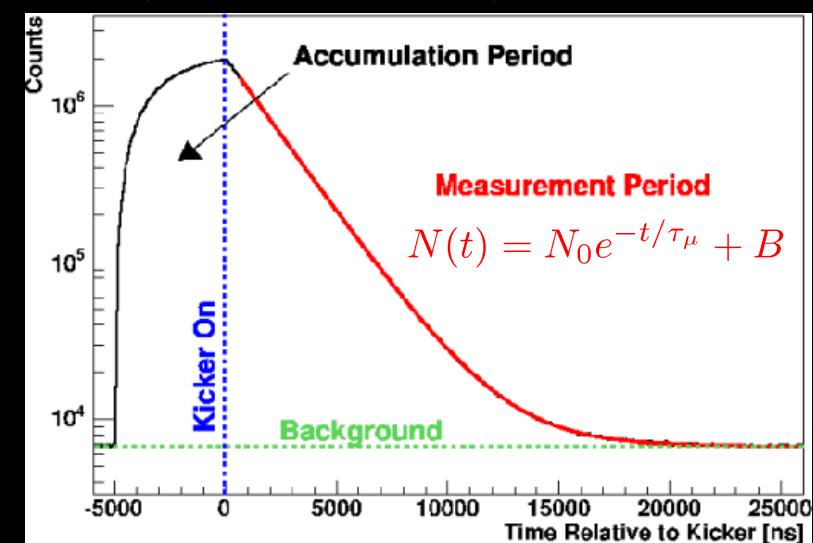
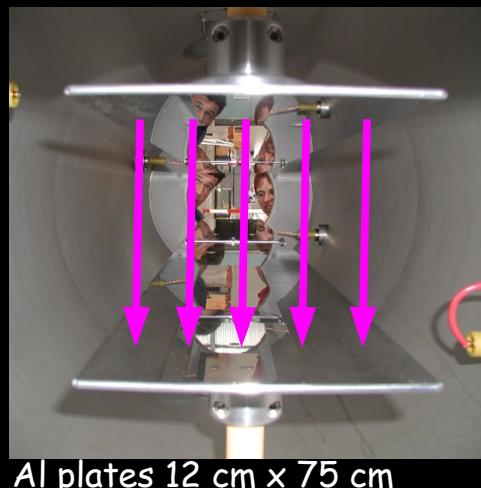
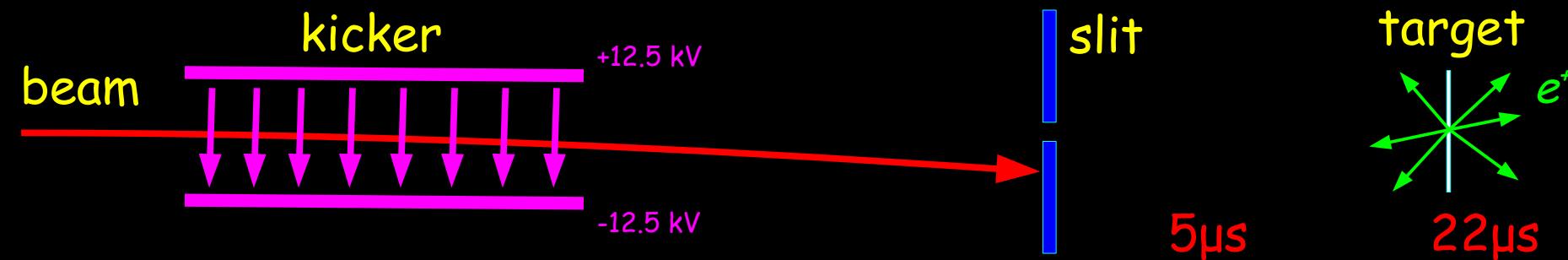
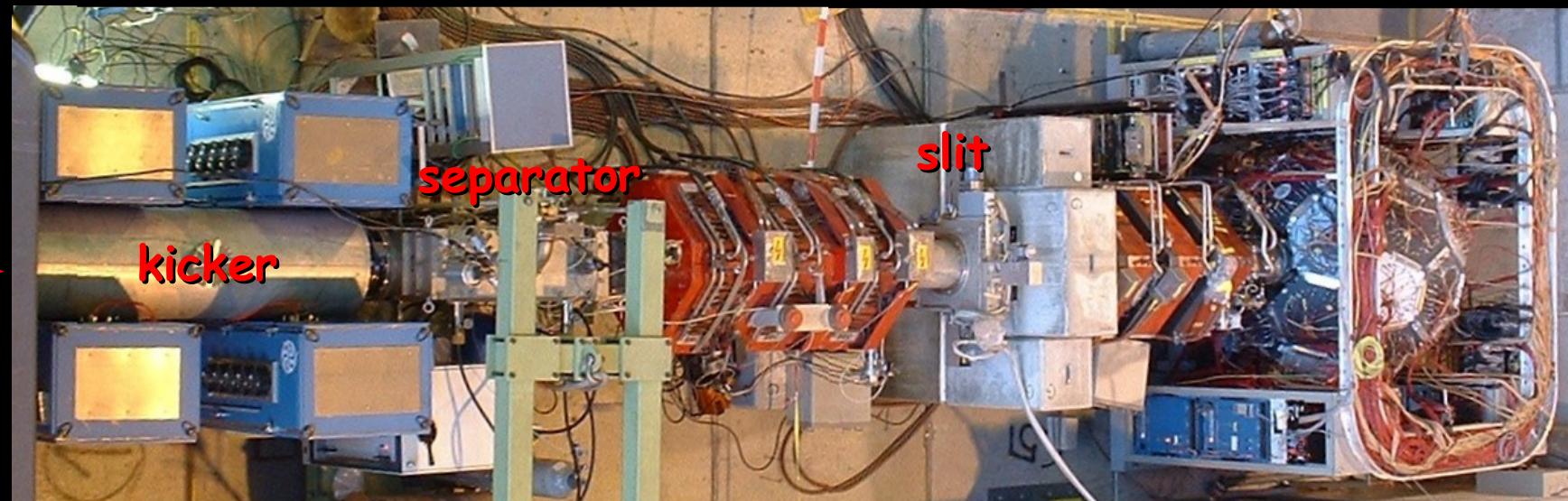


Al plates 12 cm x 75 cm

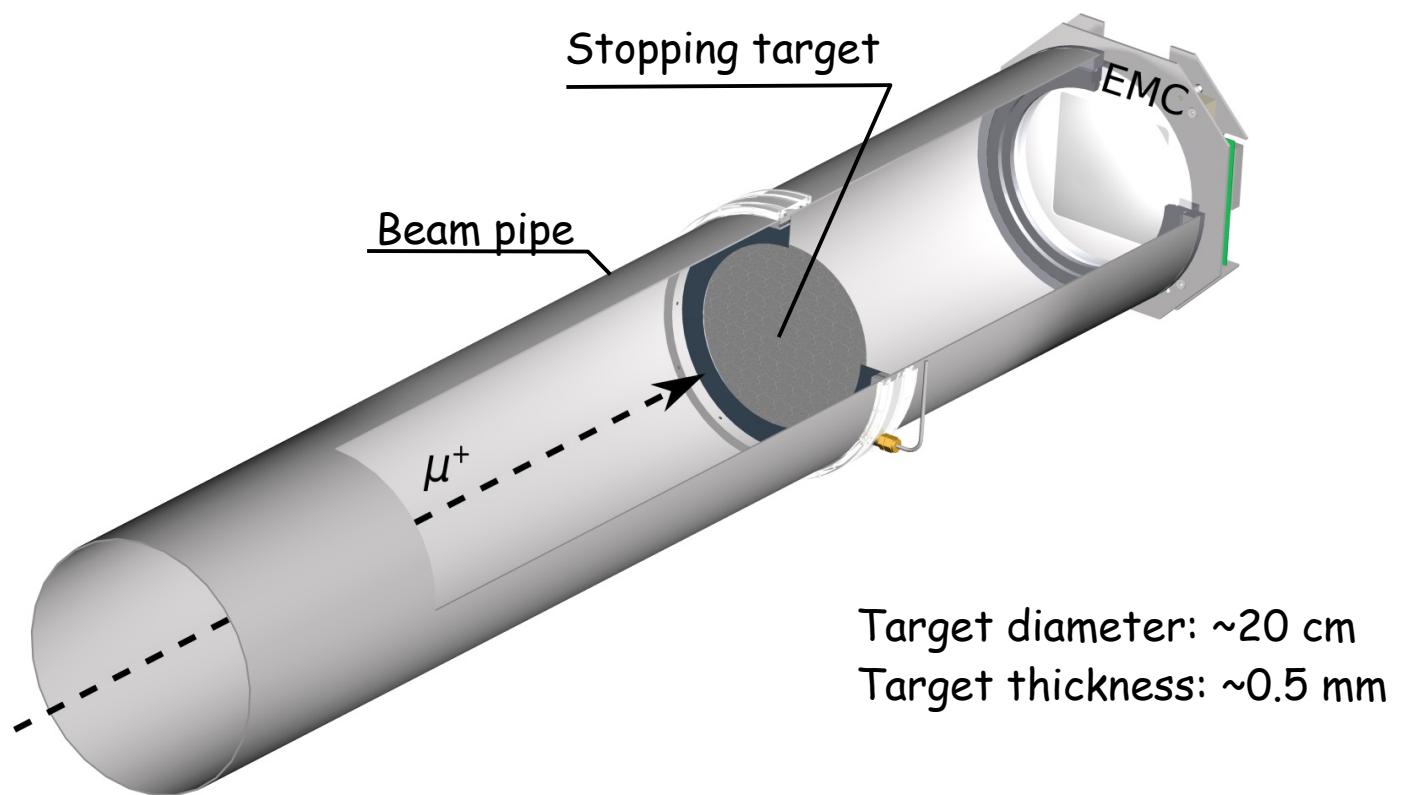


$$N_{\text{in}}(t) = \nu_{\mu} \tau \left( 1 - e^{-t/\tau_{\mu}} \right)$$

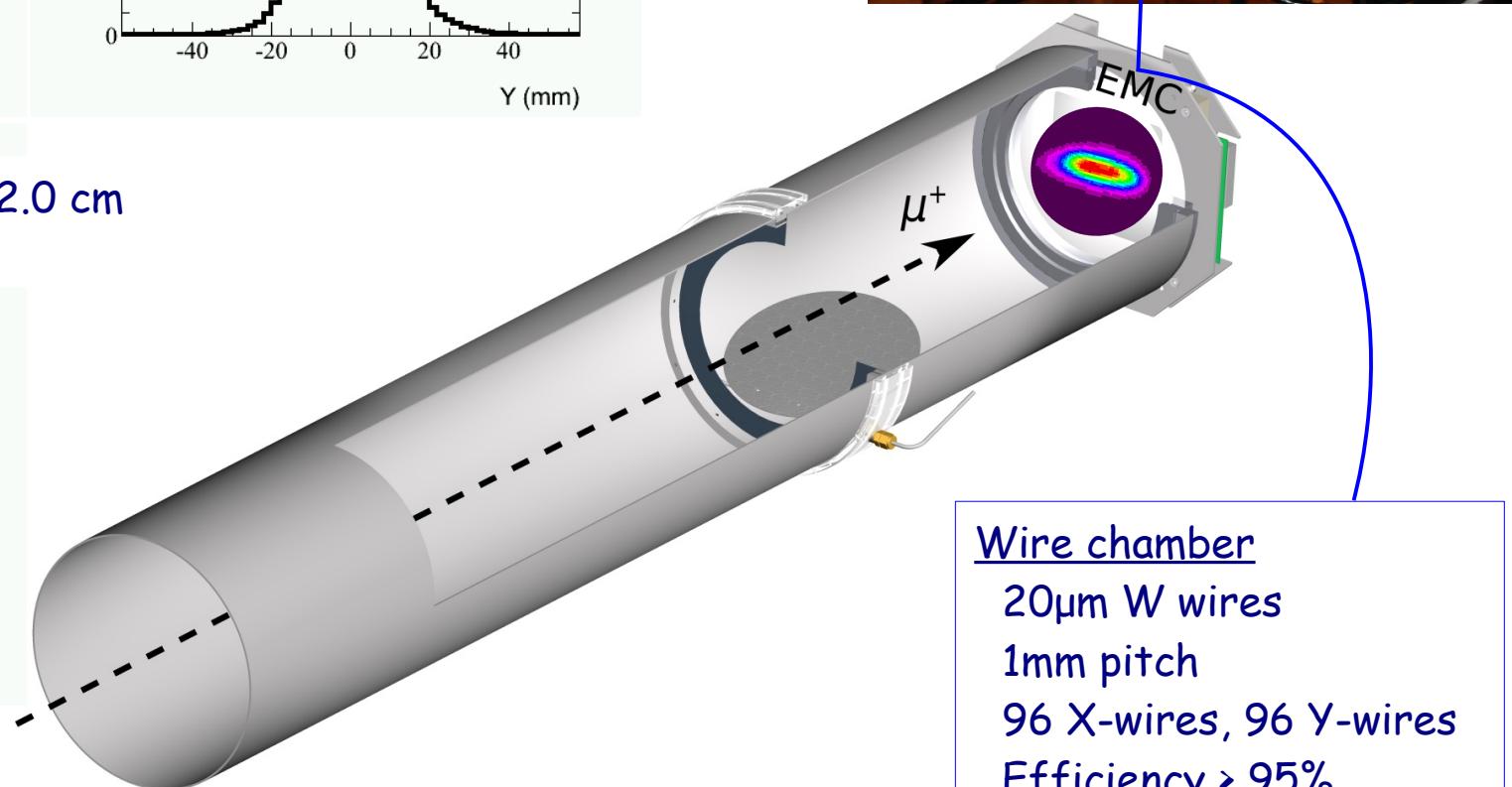
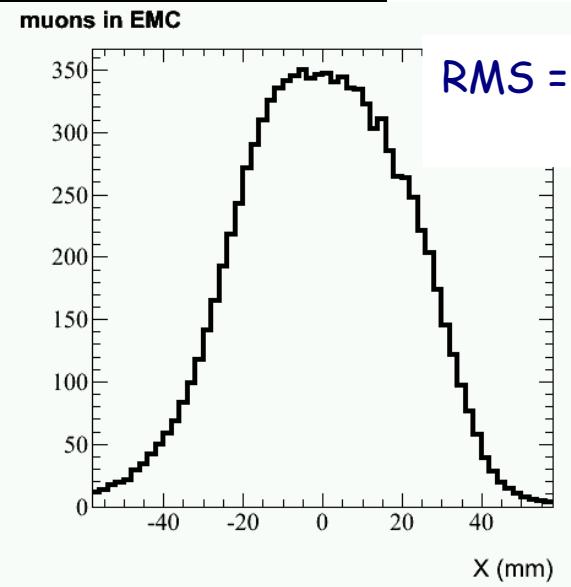
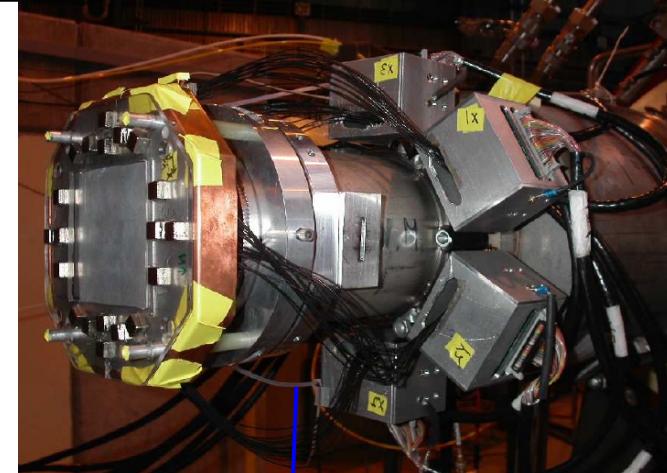
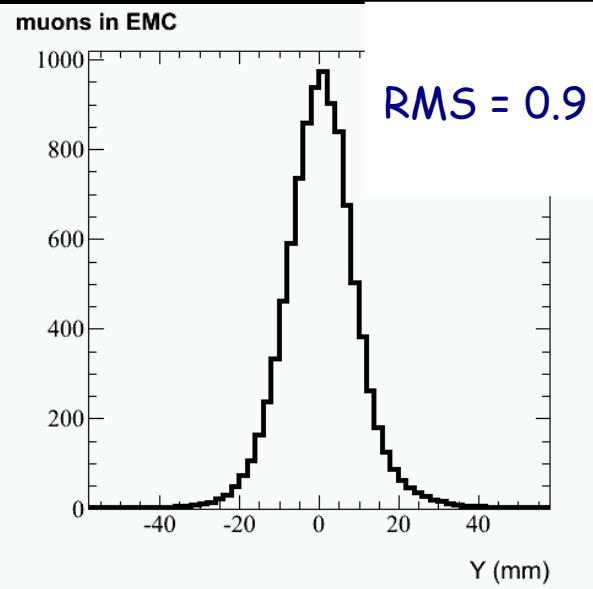
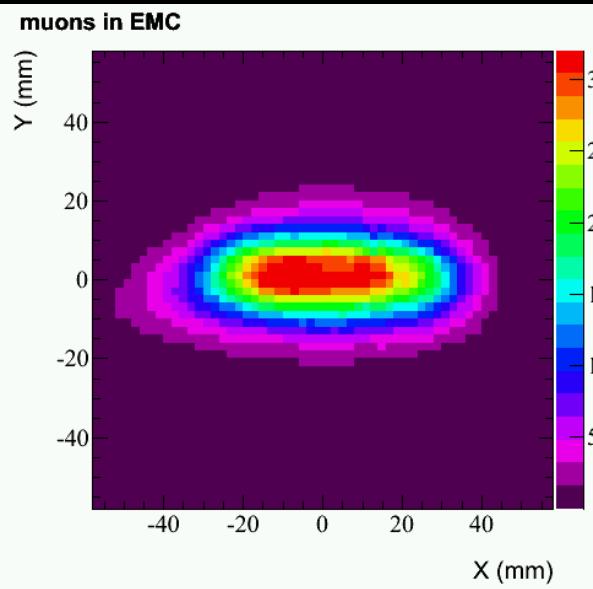
# The Kicker is used to create a pulsed beam

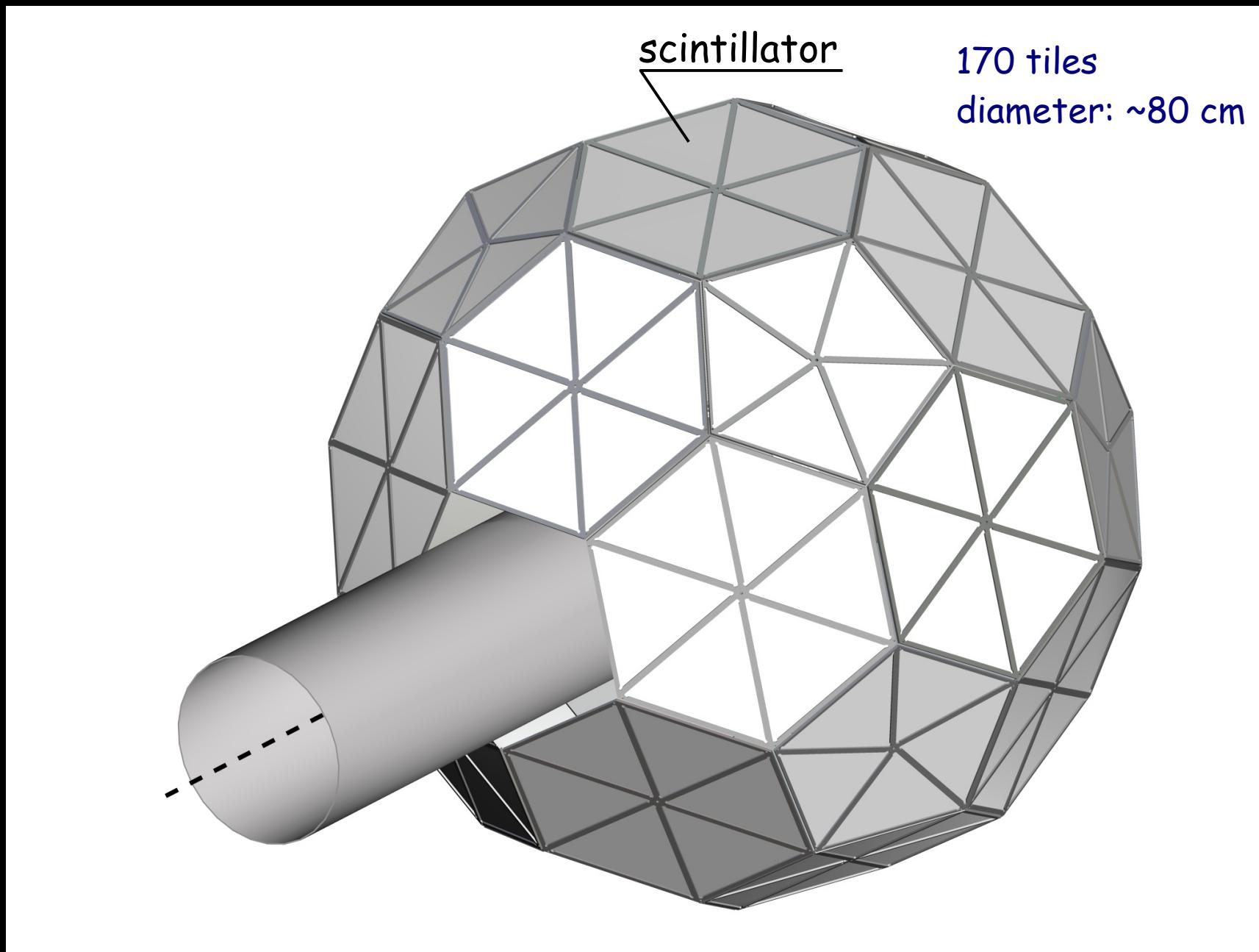


# Stopping target



# The target was opened once per day to monitor the beam

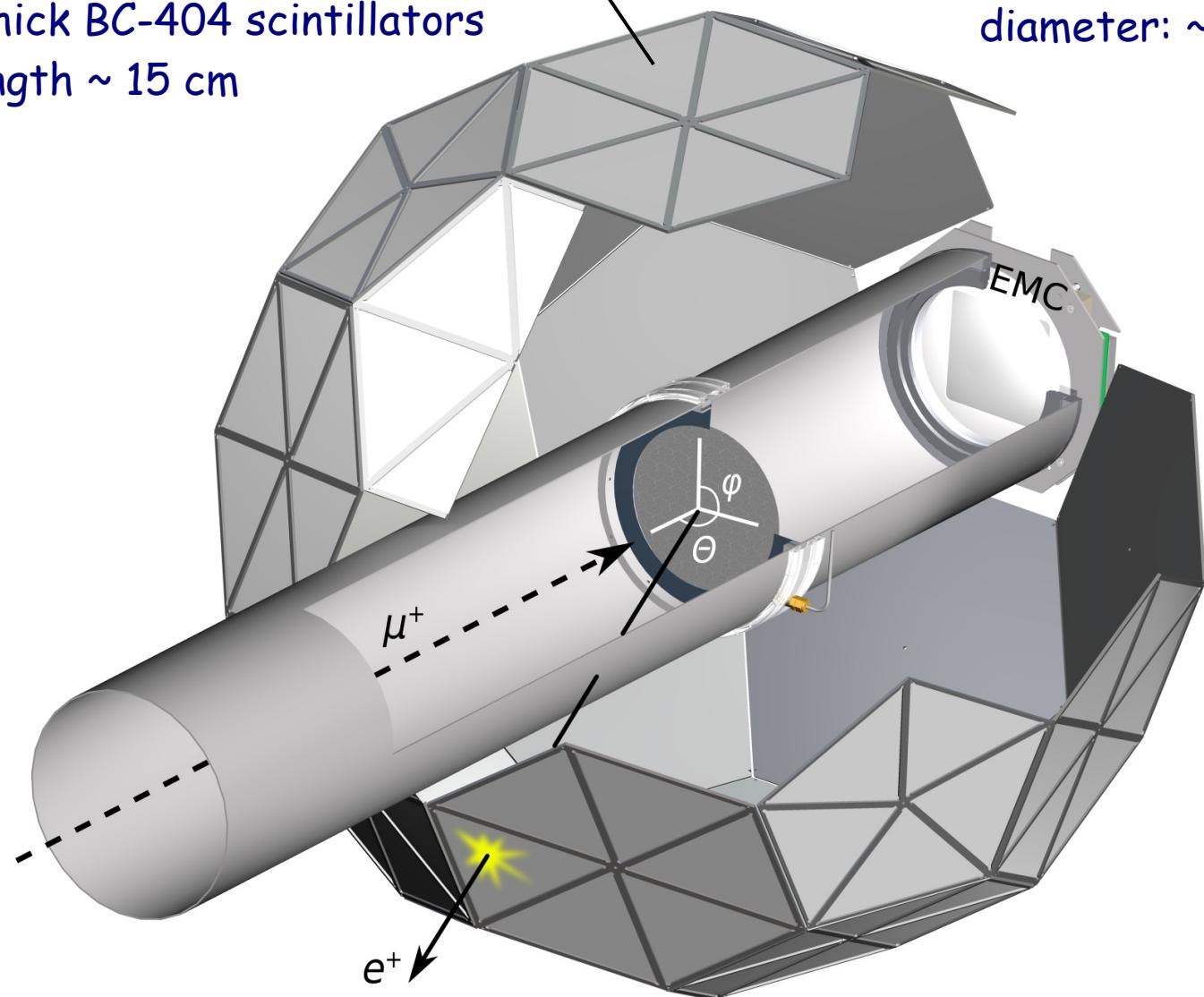


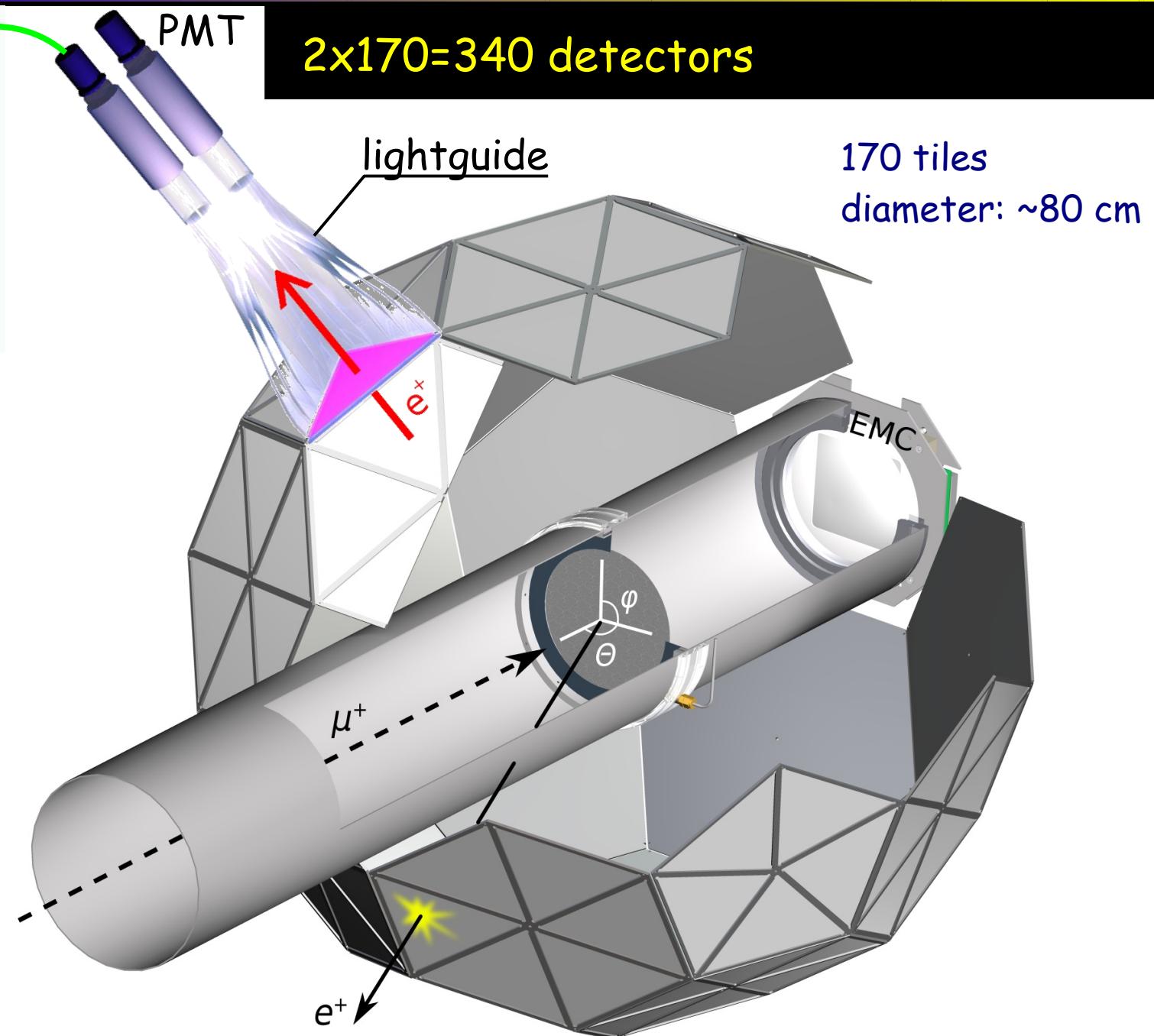
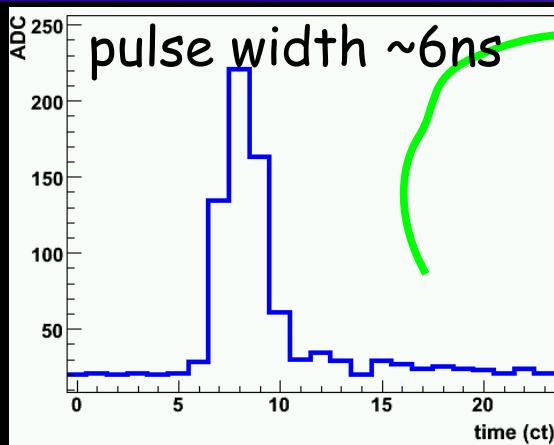


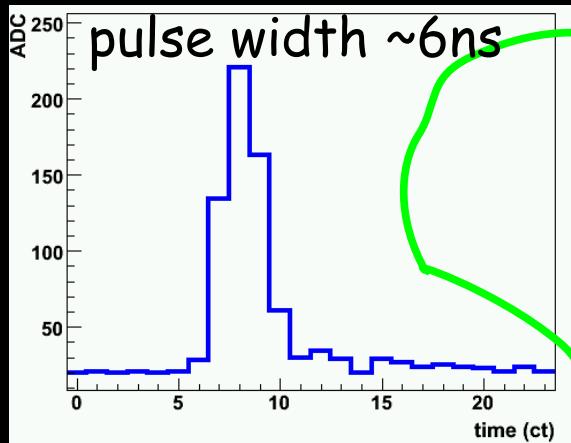
one tile is a pair of  
3-mm thick BC-404 scintillators  
base length  $\sim 15$  cm

scintillator

170 tiles  
diameter:  $\sim 80$  cm

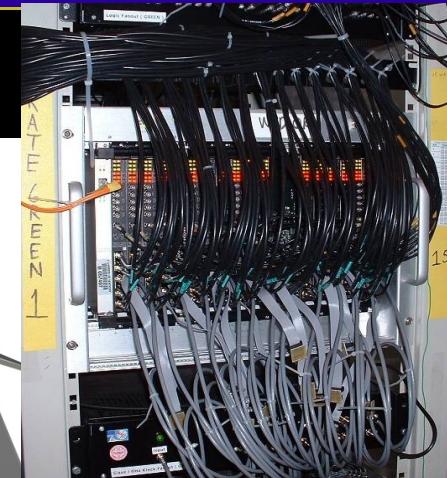
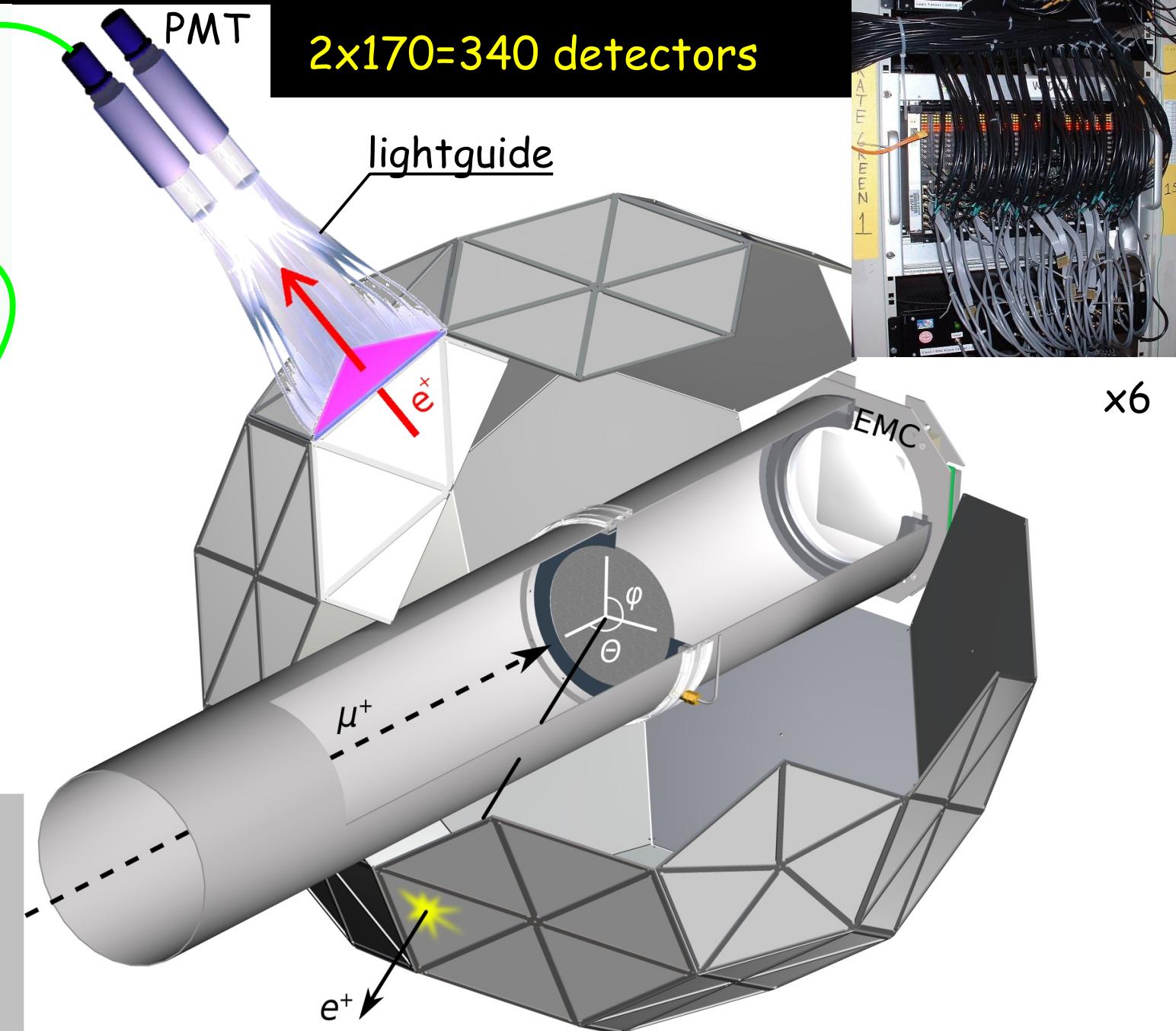




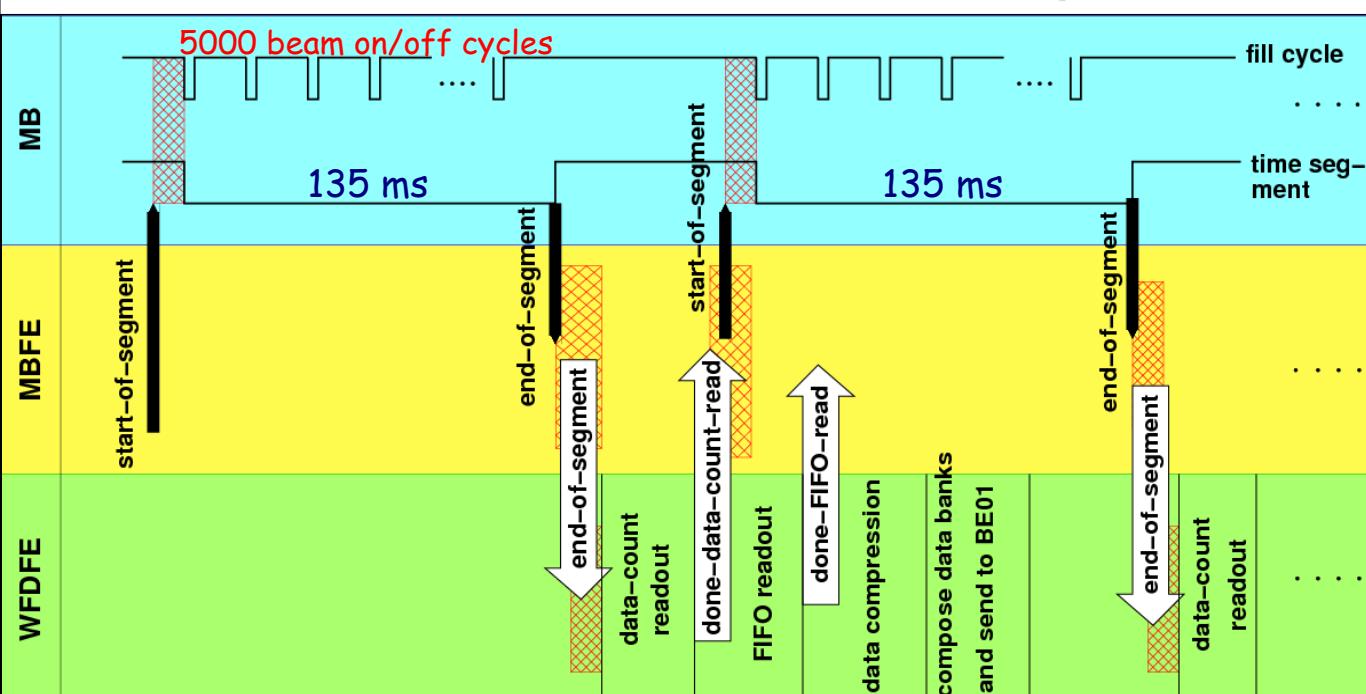
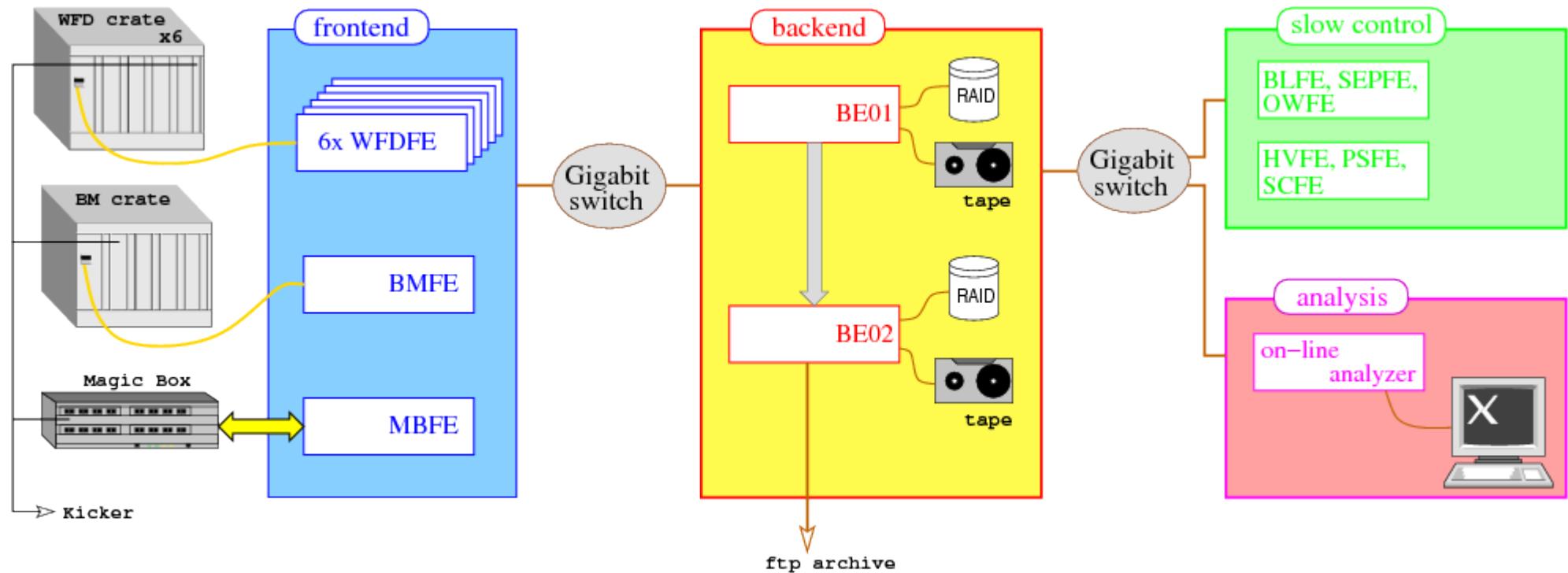


Boston 500MHz WFD

- 4 channels/board
- Inner and outer tile as well as opposite tiles on one board
- 64 bit wide data bus to VME
- total rate ~40 MB/s
- $\sim 2 \times 10^{12}$  pulses recorded



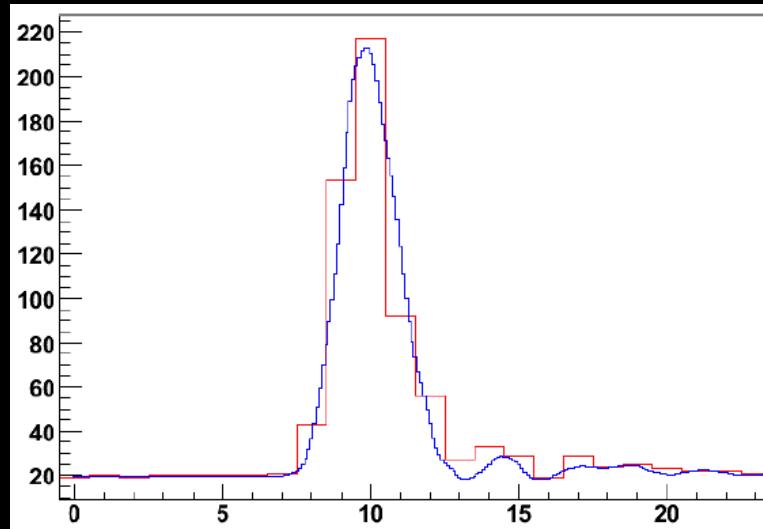
# Data Acquisition System (DAQ)



1 ppm  $\rightarrow 10^{12}$  muon decays

- ~50 MB/s raw data stream
- ~ 130 TB of data collected
- deadtime-free data segments
- ~97% DAQ livetime
- On-line data compression
- Double-core Intel™ Xeon two-CPU PCs
- Multi-threading

# Data Analysis Flow



raw waveforms are fit with templates to find pulse amplitudes and times



fit for  $\tau_\mu$

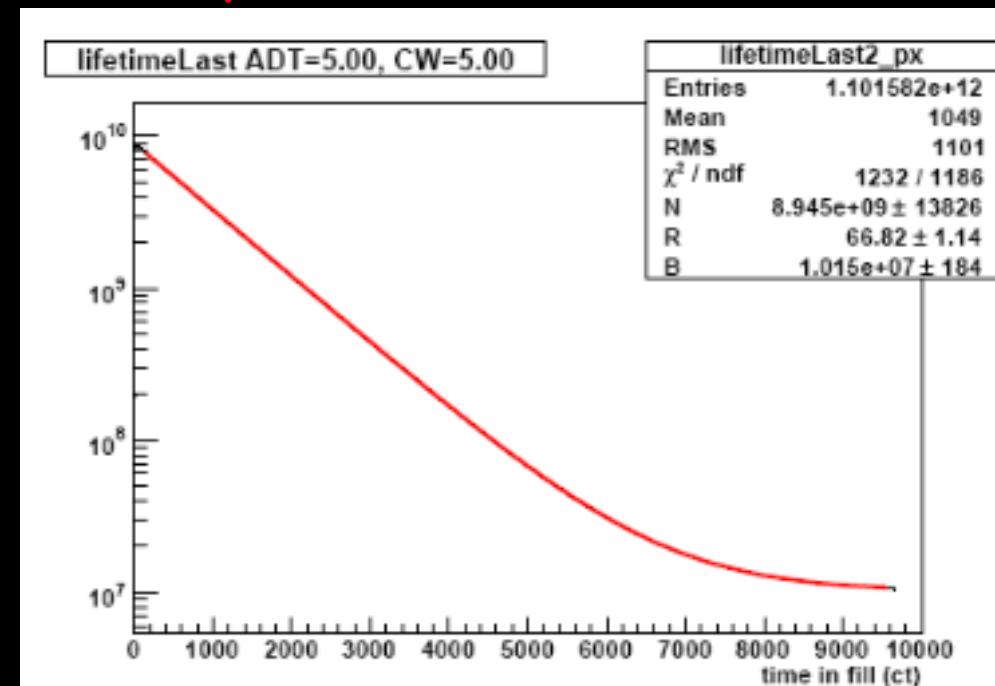
$$f(t) = N_0 e^{-t/\tau_\mu} + B$$

## Blind analysis

- Exact clock frequency kept secret
- Muon lifetime is reported is reported with the value R with units of ppm defined as

$$\tau_\mu = \tau_{\text{secret}} (1 + R/10^6)$$

only analyzer knows



check consistency,  
study systematic errors



# What can go wrong?

Early-to-late changes, for instance:

## Instrumental issues

PMT gains

Discriminator threshold walk

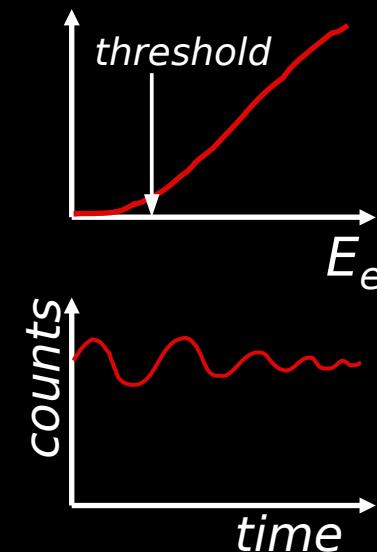
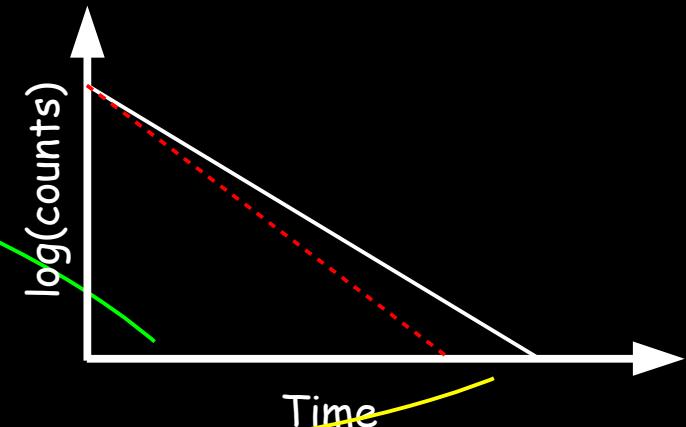
Kicker voltage sag

Pileup

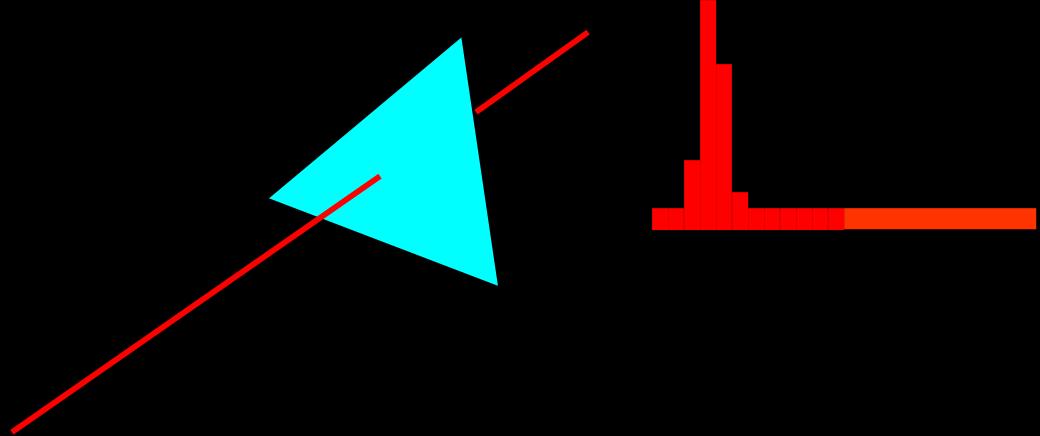
## Physics issues

Spin polarization

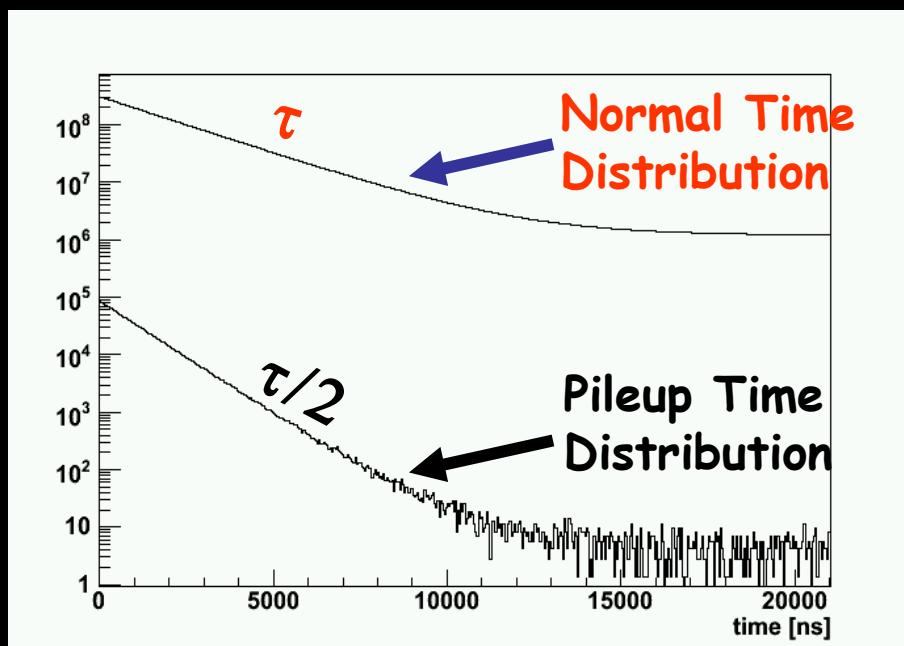
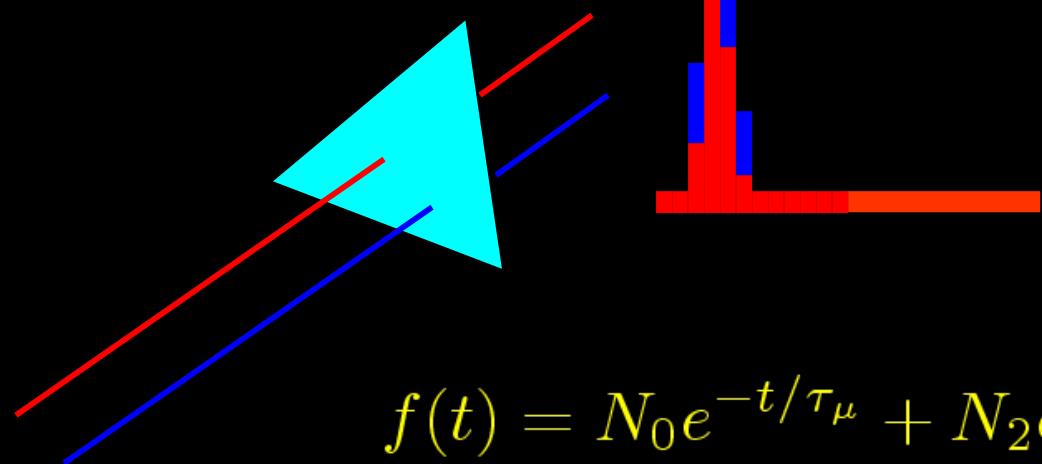
Non-flat background sources



## Pileup

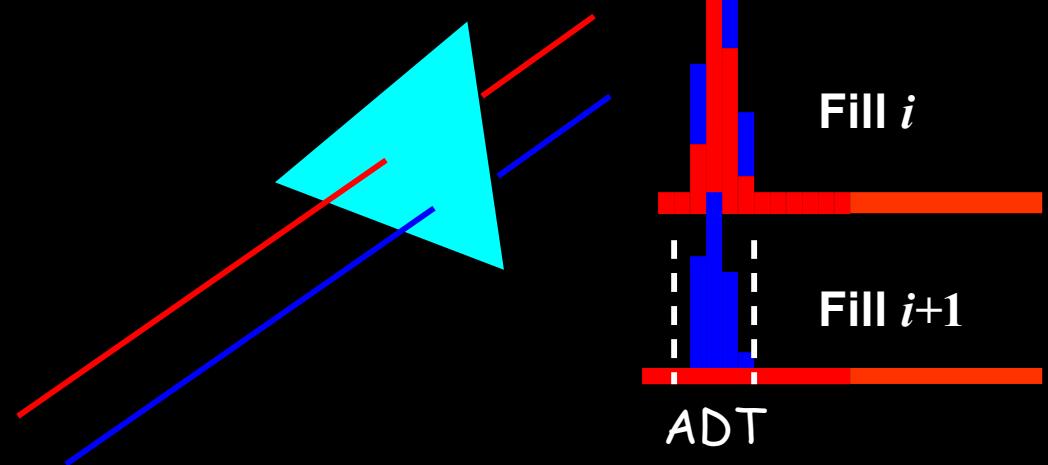


Leading order pileup to a  $\sim 5 \times 10^{-4}$  effect

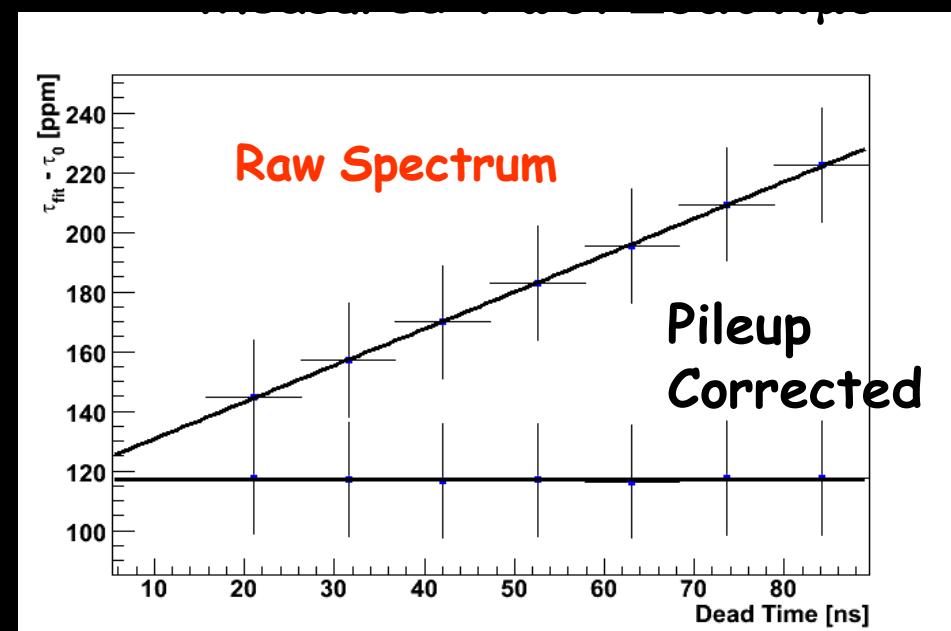
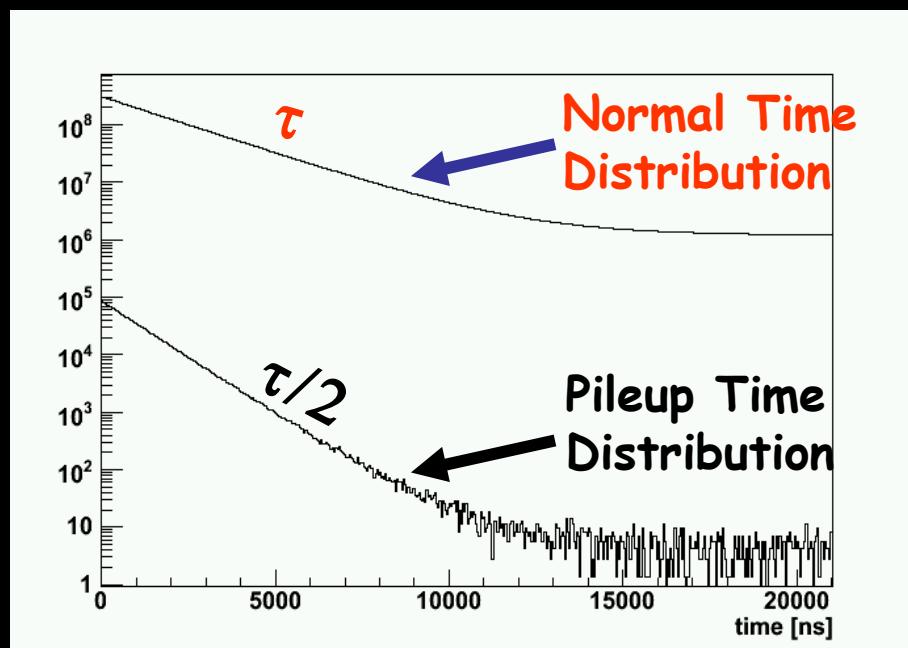


# Pileup

Leading order pileup to a  $\sim 5 \times 10^{-4}$  effect



- Statistically reconstruct pileup time distribution
- Fit corrected distribution



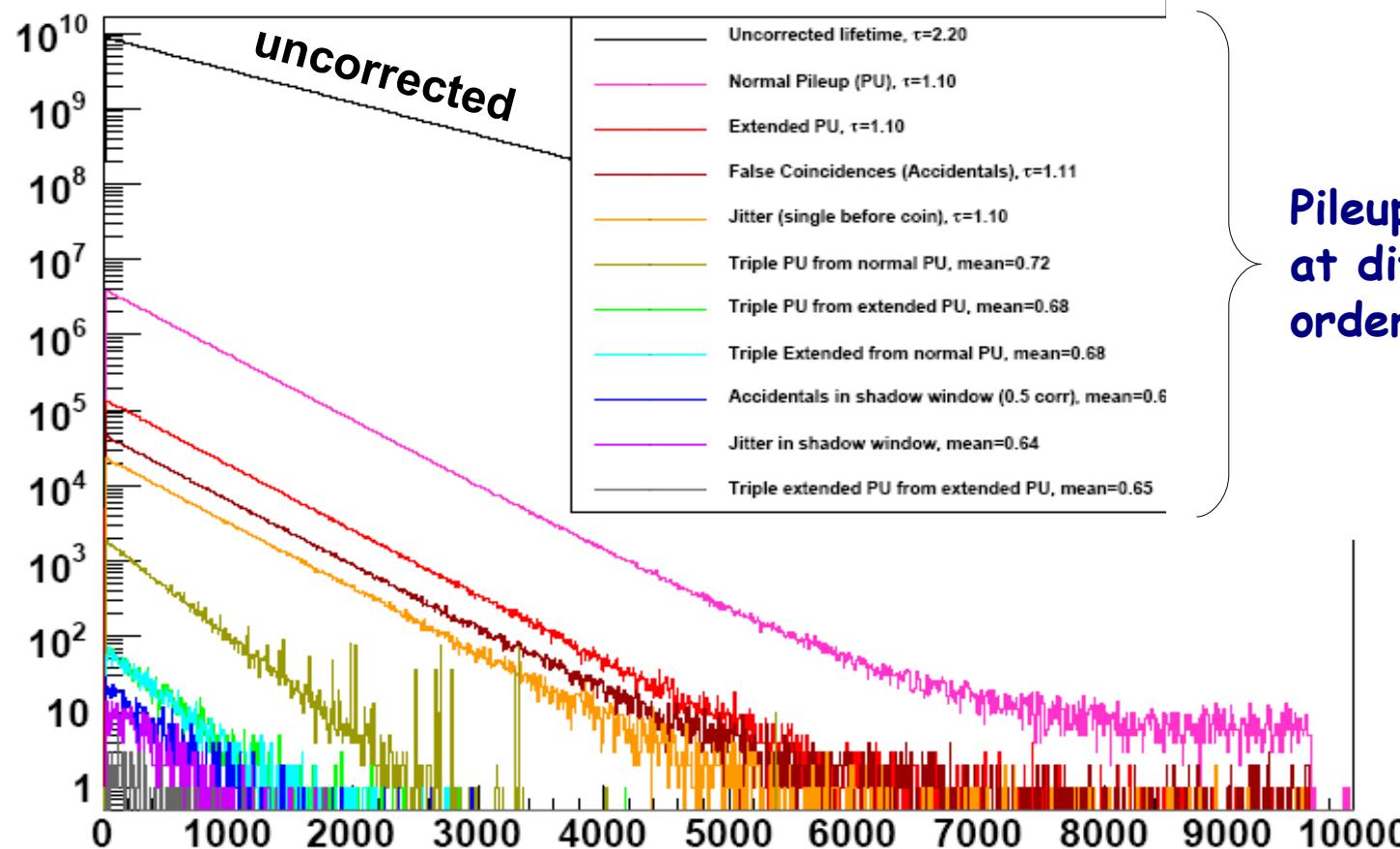
# Pileup to sub-ppm requires higher-order terms

12 ns deadtime, pileup has a  $5 \times 10^{-4}$  probability at our rates

Left uncorrected, lifetime wrong by 100's of ppm

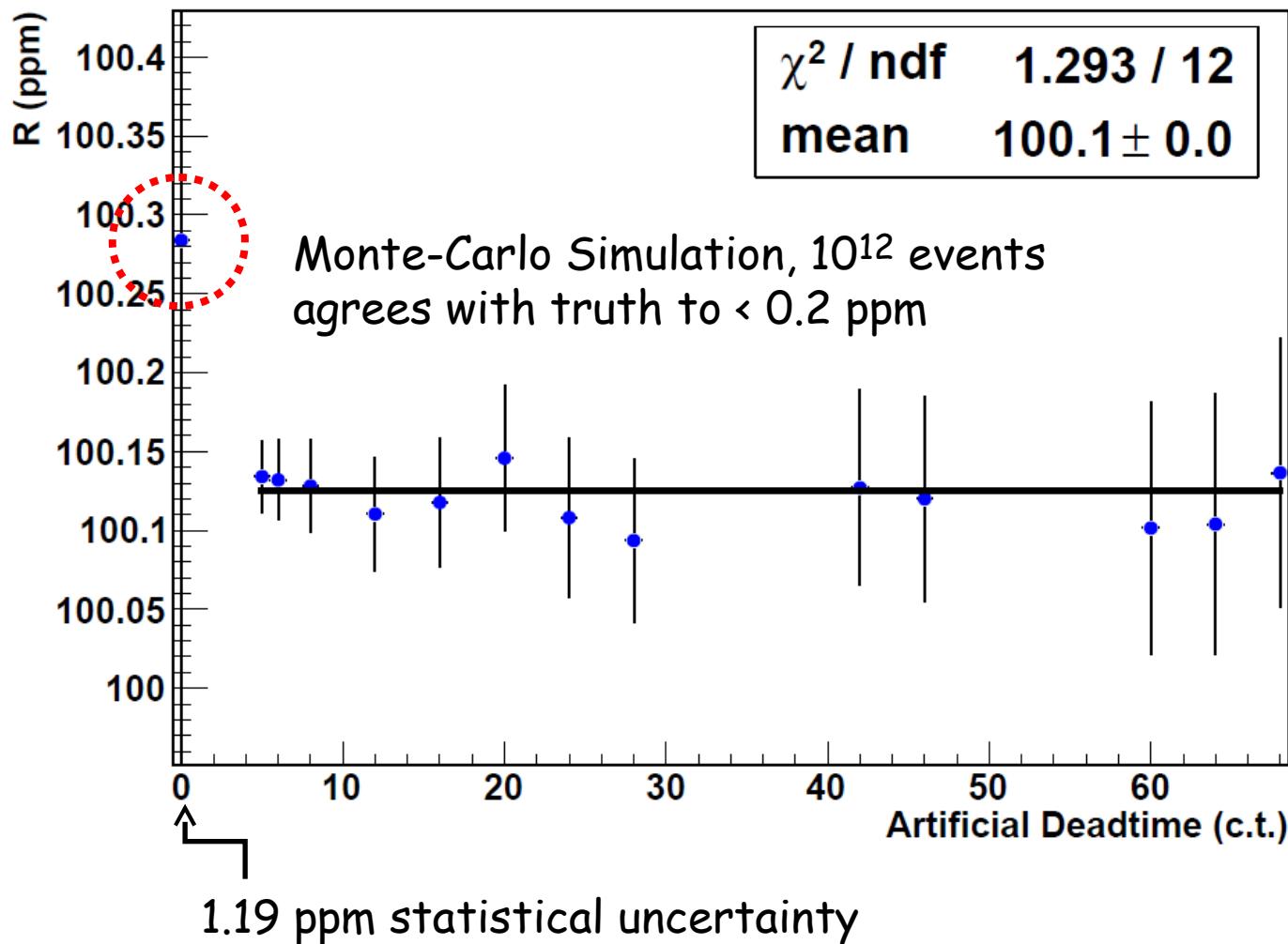
Proof of procedure validated with detailed Monte Carlo simulation

lifetimeLast ADT=5.00, CW=5.00

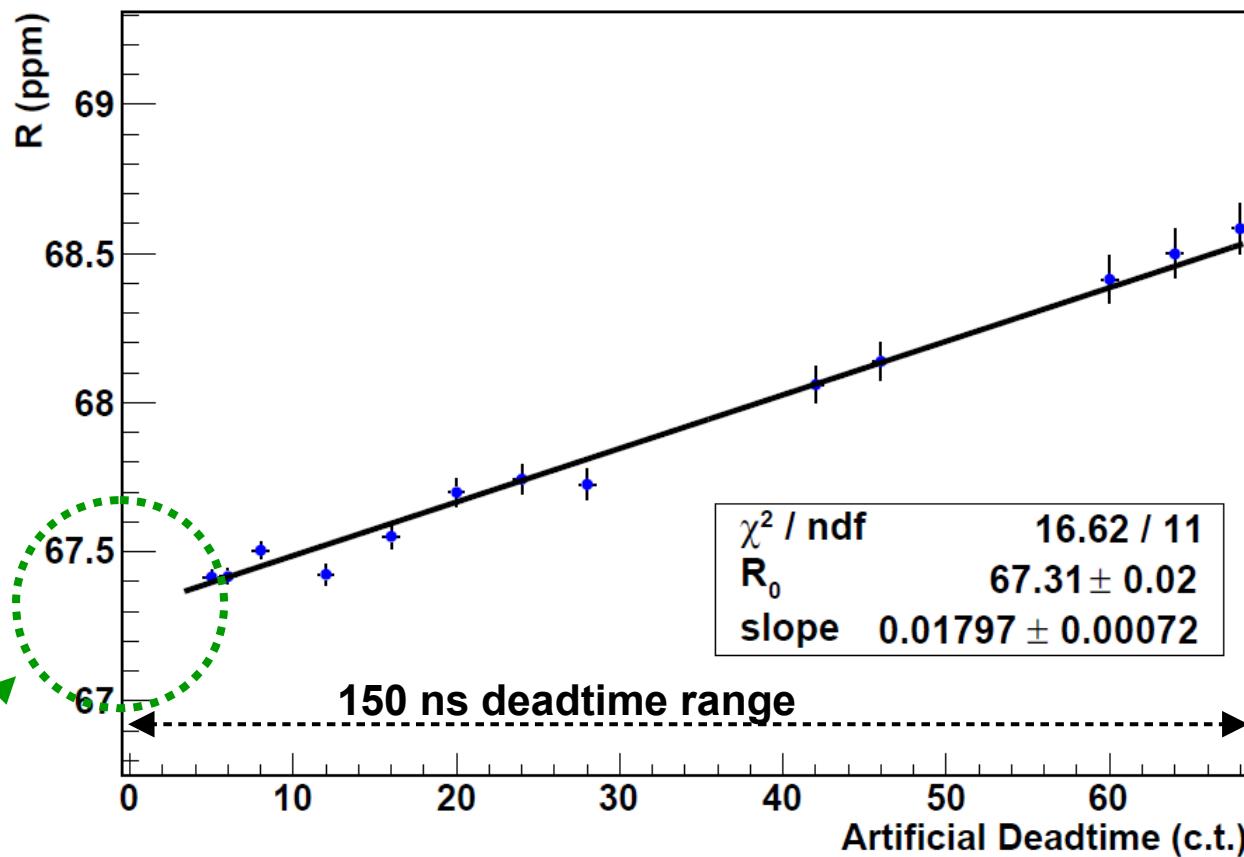


D. M. Webber

# The pileup corrections were tested with Monte-Carlo.



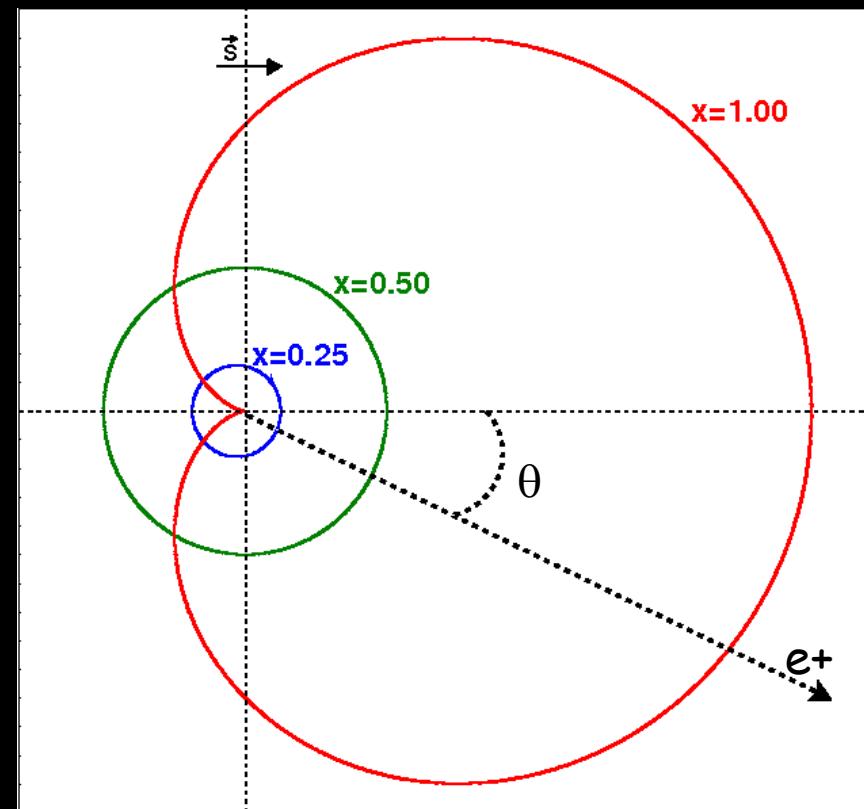
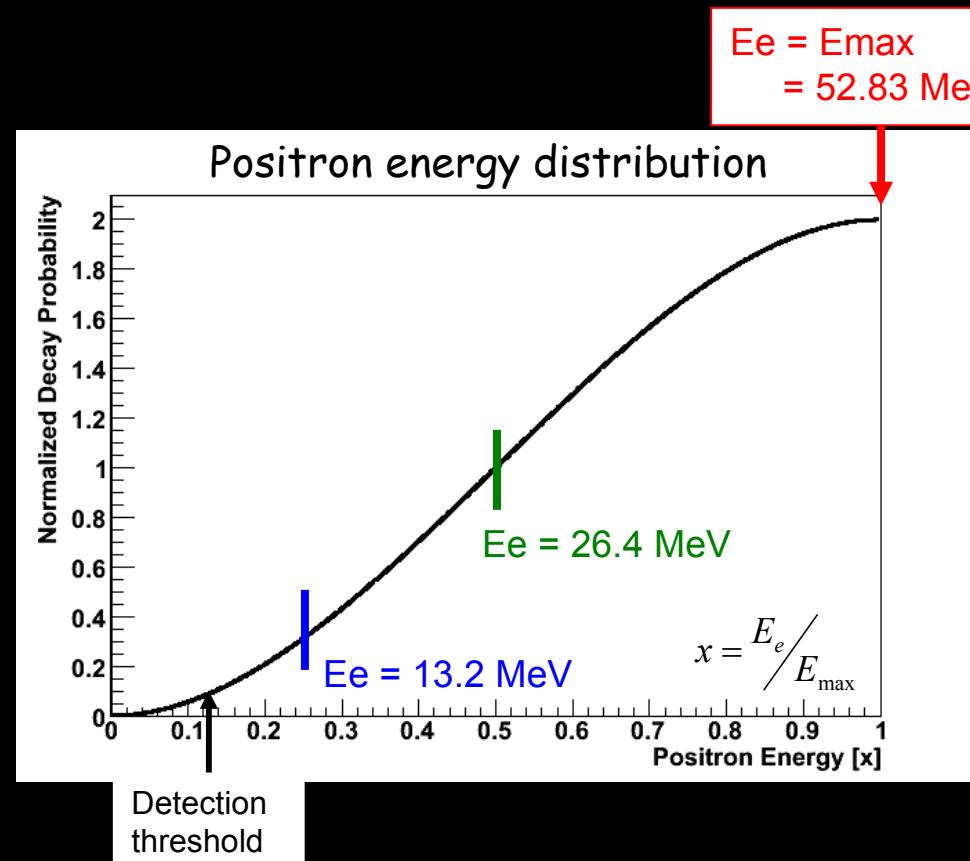
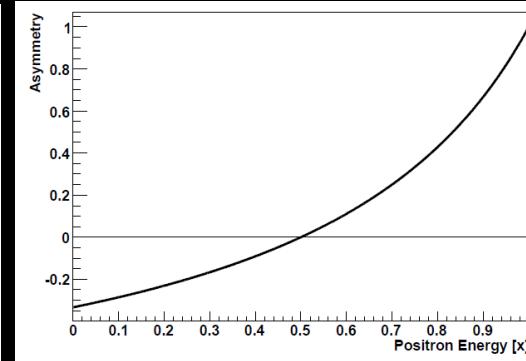
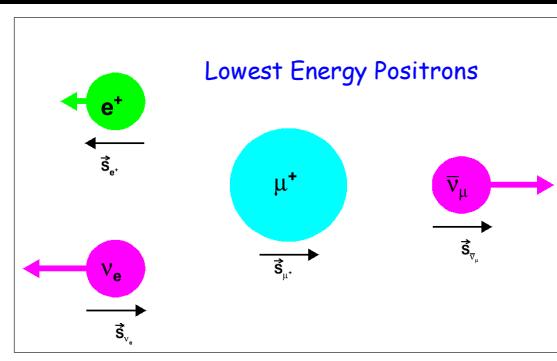
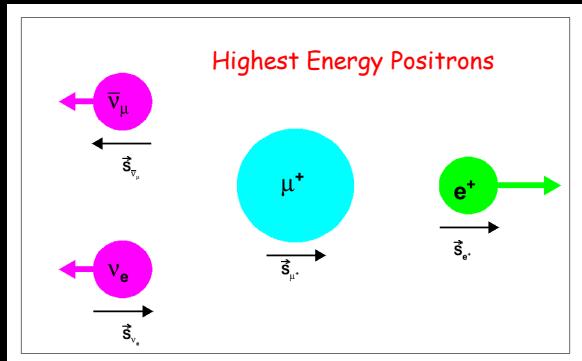
A slope exists due to a pileup undercorrection



Extrapolation to 0 deadtime is correct answer

Pileup Correction Uncertainty: 0.2 ppm

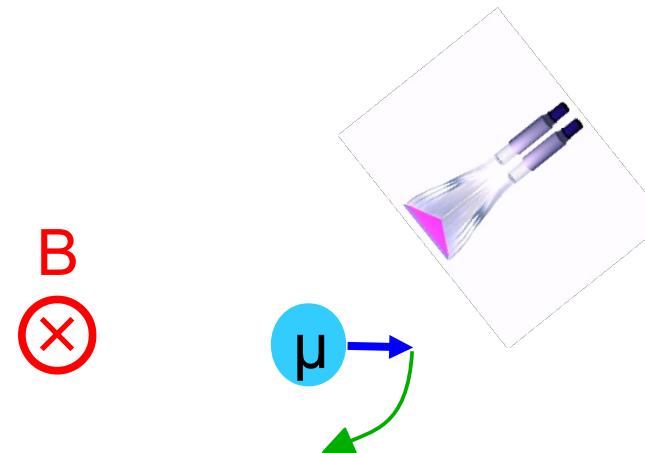
The decay positron energy and angular distributions are not uniform, resulting in position dependent measurement rates.



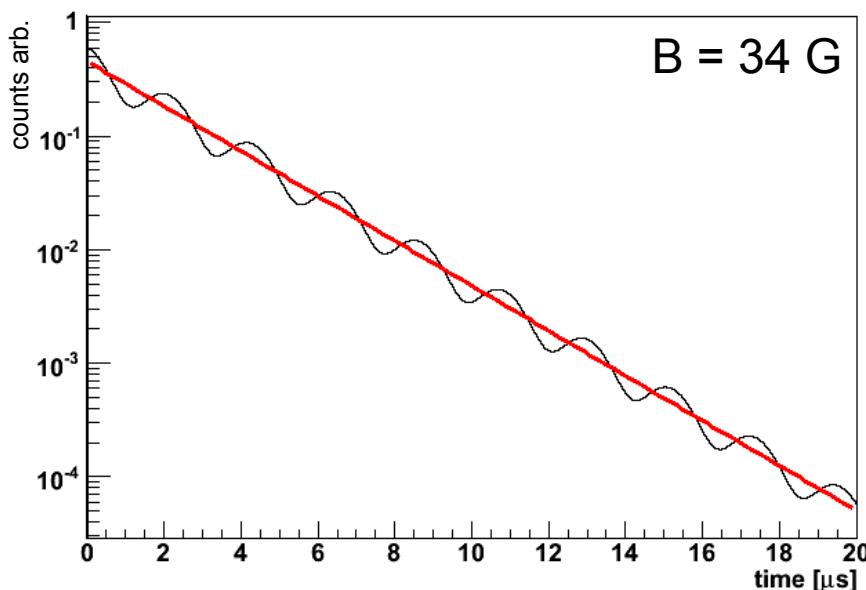
$$\omega = g_\mu \frac{eB}{2m_\mu c}$$

$$g_\mu \approx 2$$

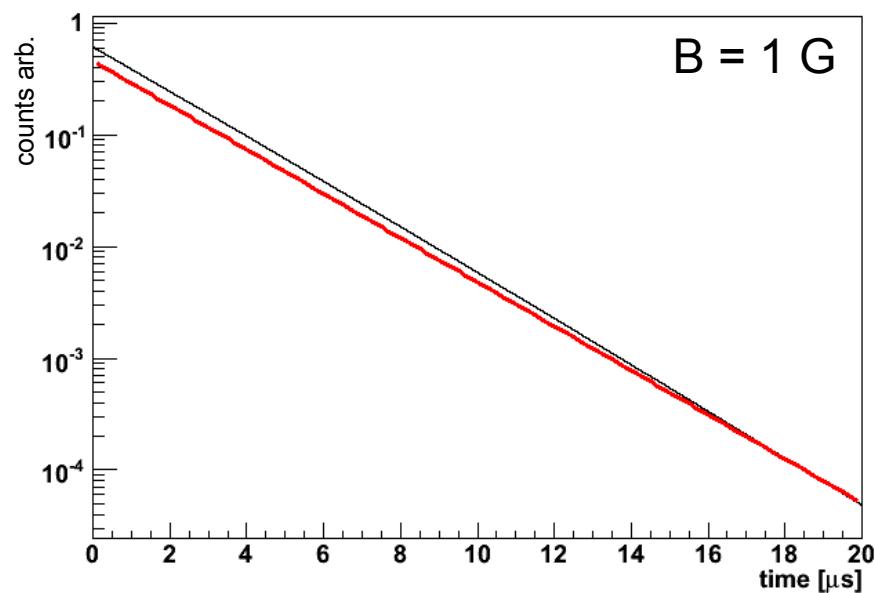
$$\omega \sim 135 \text{ MHz/T}$$



$$N(t) = N_0 e^{-t/\tau_\mu} [1 + AP_2 \cos(\omega t + \phi_0)]$$

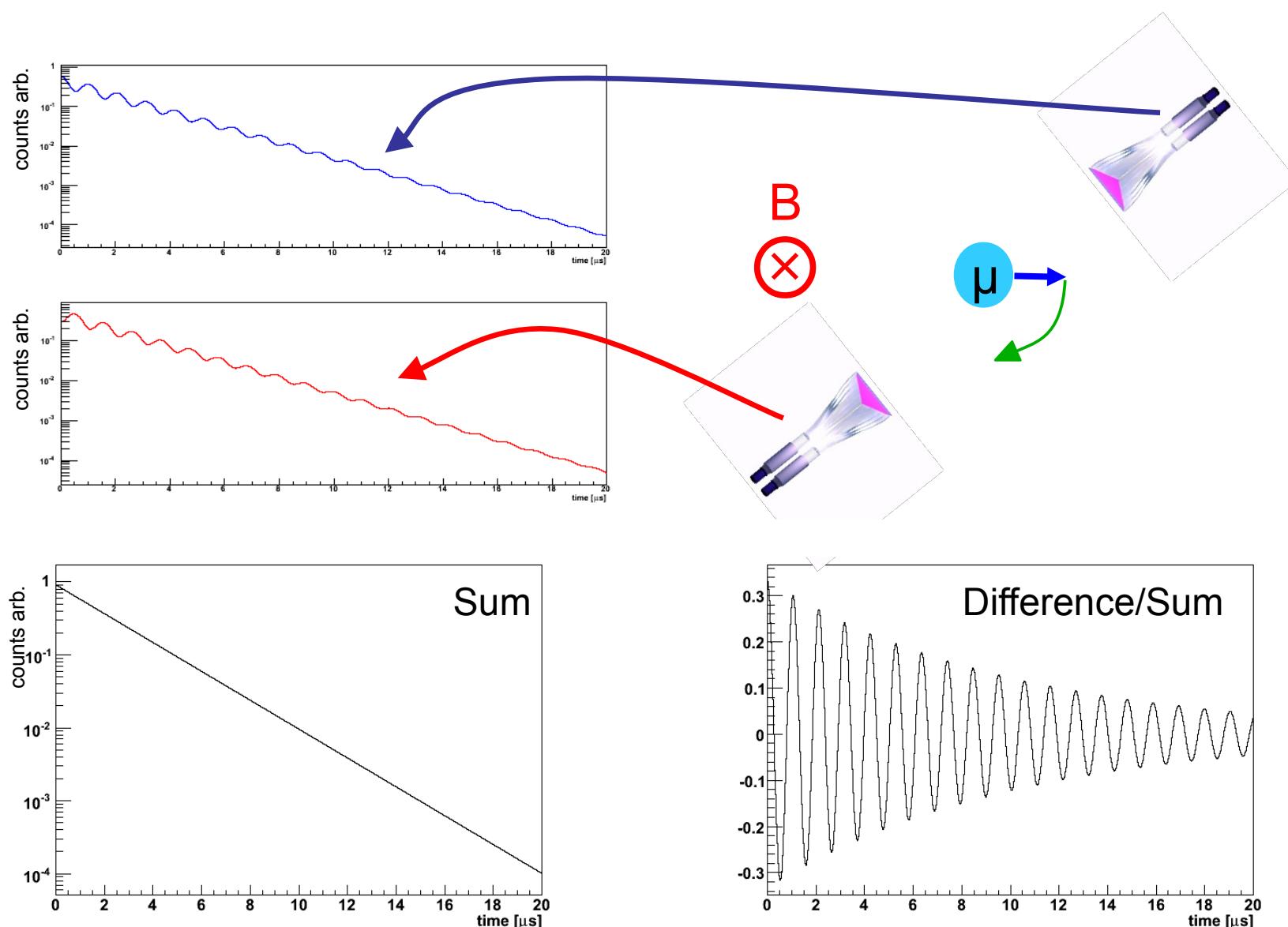


This oscillation is easily detected



This oscillation is not easily detected  
and systematic errors may arise

# $\mu$ SR signal in opposite detectors



The sum cancels  $\mu$ SR effects

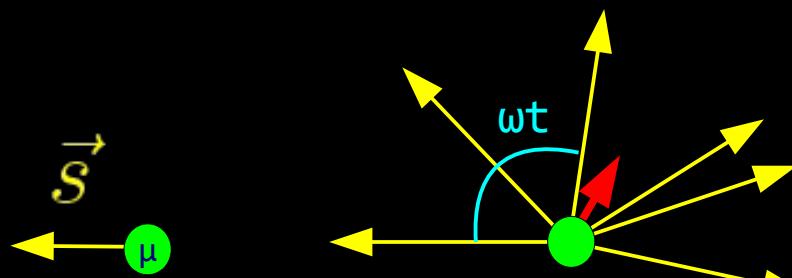
The difference accentuates  $\mu$ SR effects

- Dephasing
- Polarization destroying targets

# Dephasing

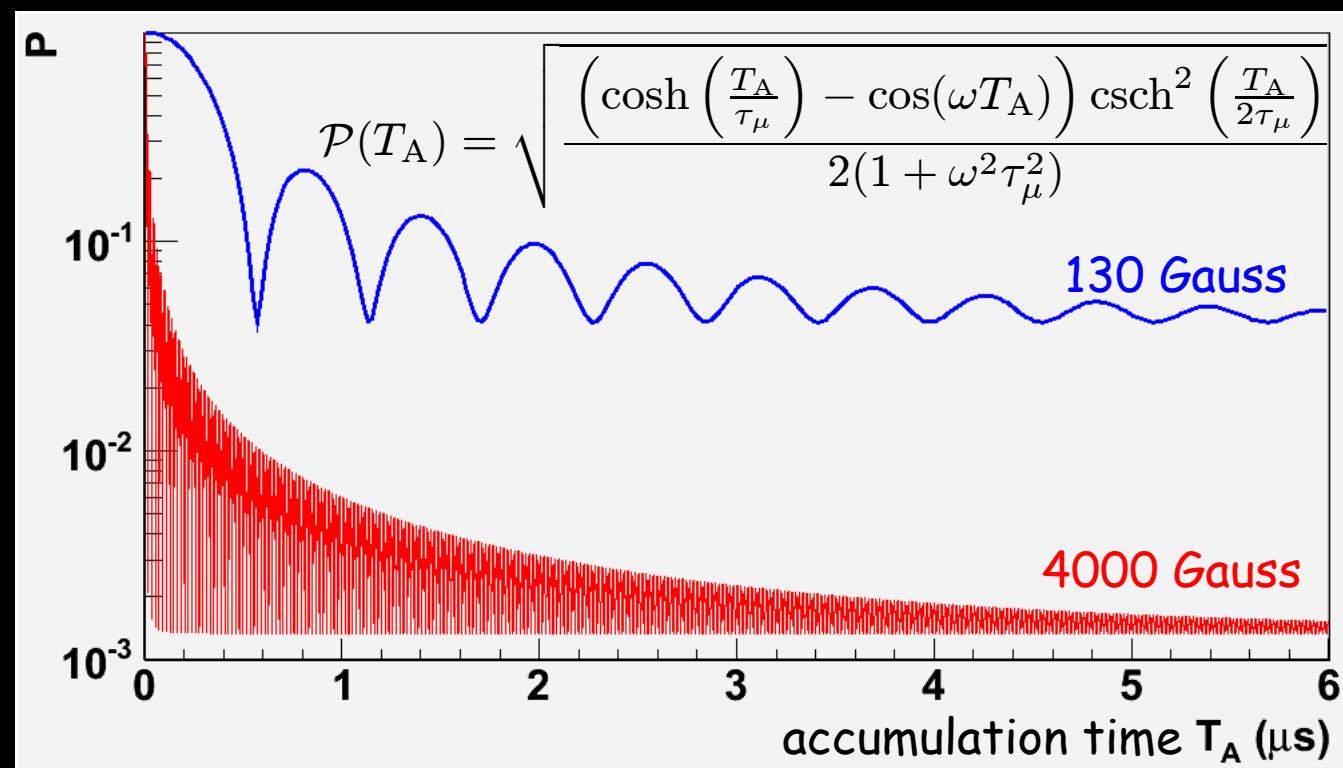
Muons arrive randomly during  $5\mu\text{s}$  accumulation period

B  
⊗



Polarization of a muon ensemble

$$\vec{\mathcal{P}}(t) = \frac{1}{n} \sum^n \vec{s}(t)$$



# Muon stopping targets

## 2006

### Arnokrome-3 (AK-3) target

(~28% chromium, ~8% cobalt, ~64% iron)  
0.4 T transverse field rotates muons with  
18 ns period

Muons precess by 0 to 350 revolutions  
**DEPHASED** small ensemble avg.  
polarization

## 2007

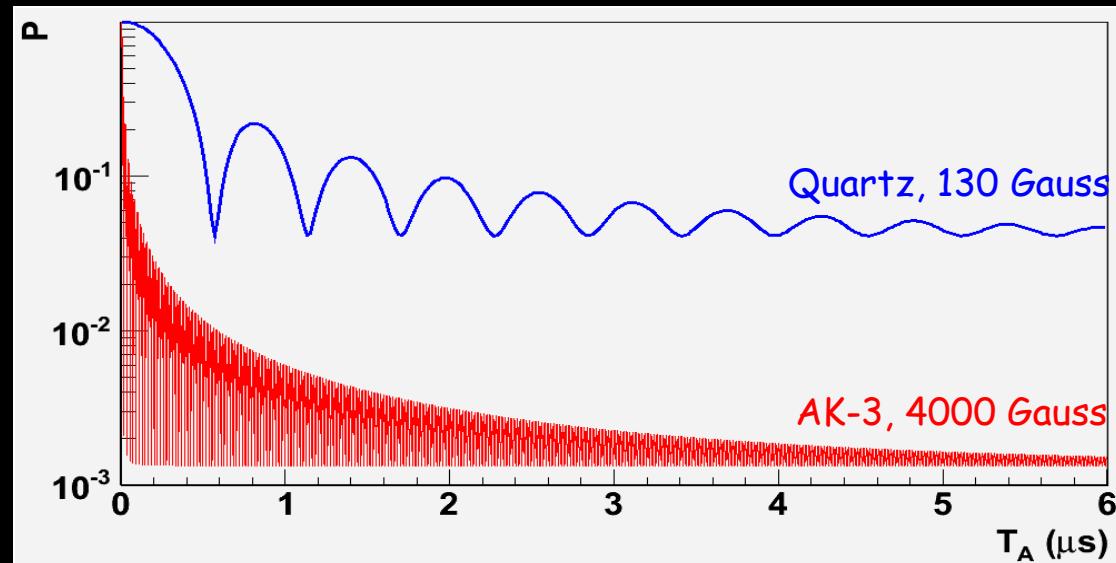
### Crystalline quartz target

#### **90% muonium formation**

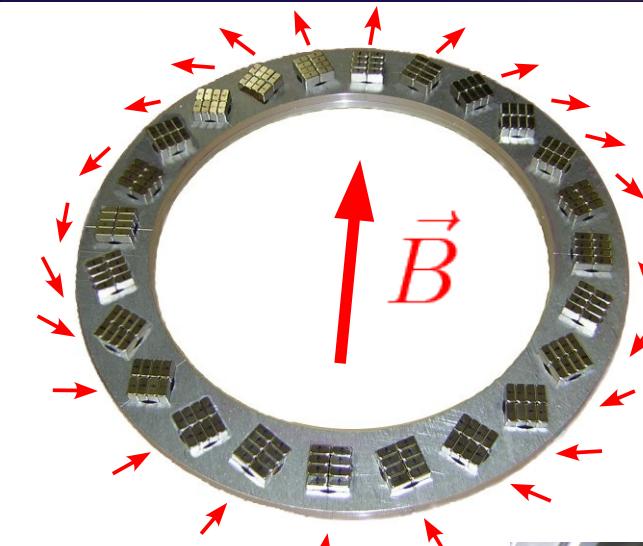
- 50% depolarization (Mu in singlet state)
- Fast precession of Mu in triplet state

#### **10% "free" muons**

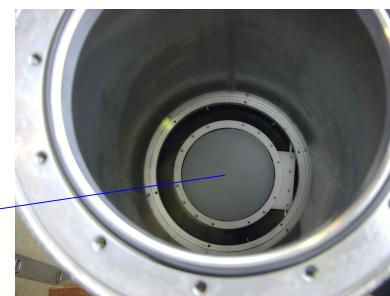
- Noticeable precession
- Relaxation of longitudinal polarization



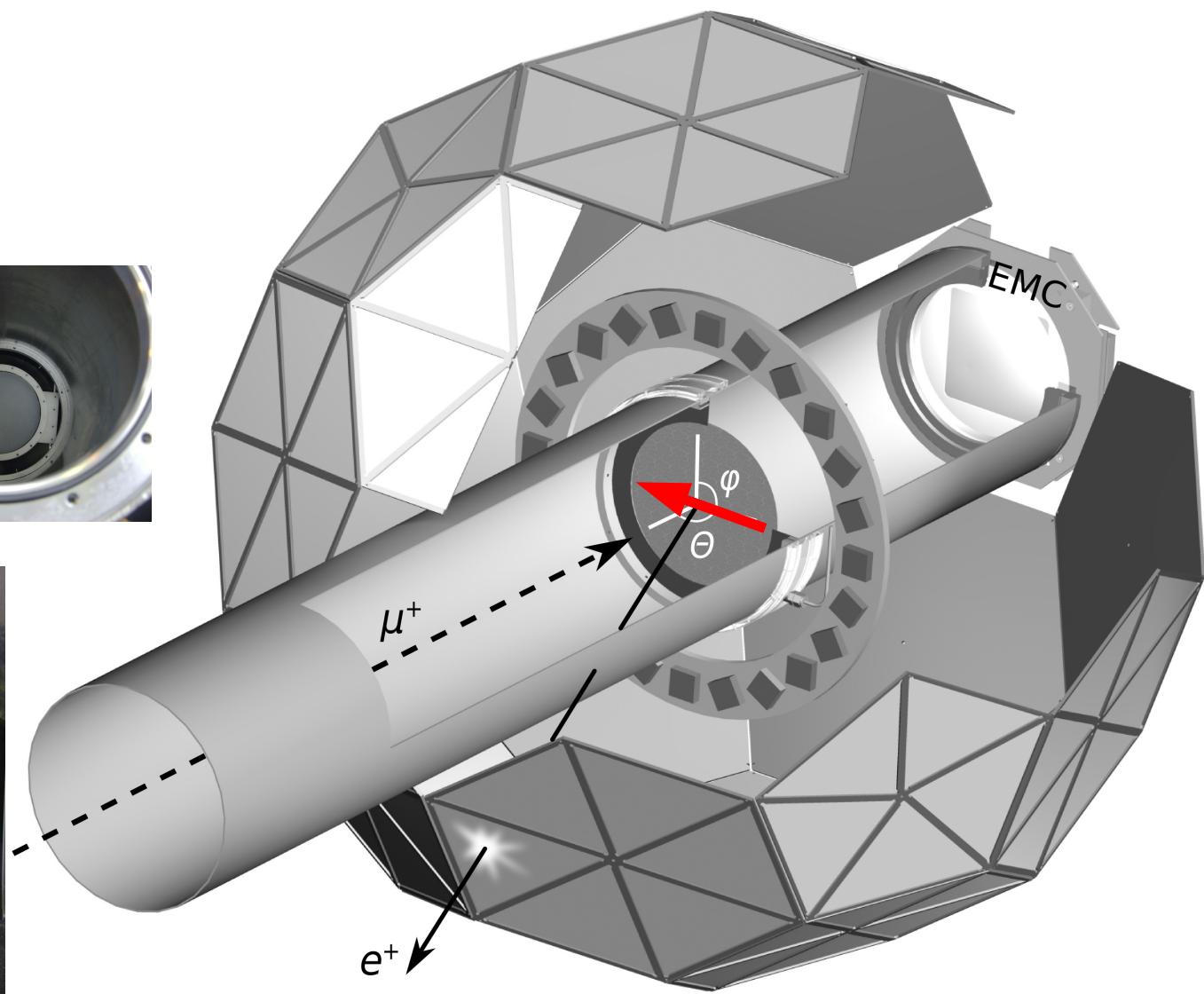
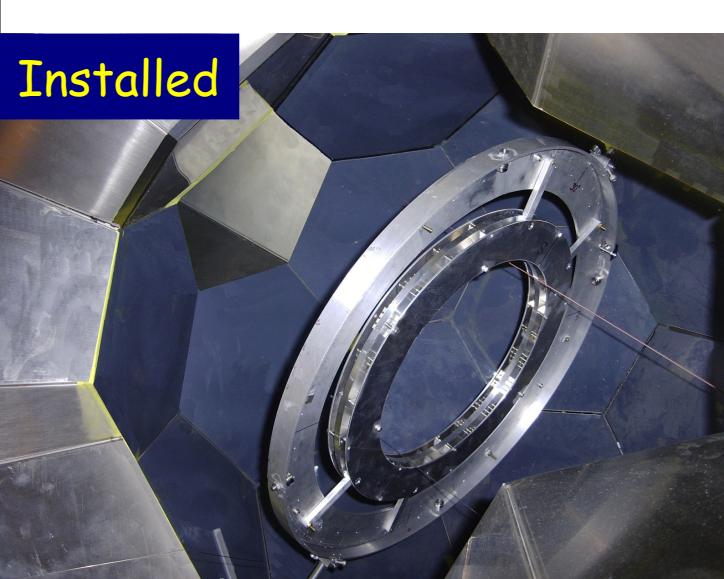
# $\mu$ SR magnet for Quartz target



Halbach array of permanent magnets to produce  $\sim 130$  G field in the target plane



Quartz target



# Two targets, two analysis strategies

## AK-3

(strongly suppressed  $\mu$ SR)

Sum time histograms from all detectors  
and fit for  $\tau_\mu$

## Quartz

(noticeable  $\mu$ SR)

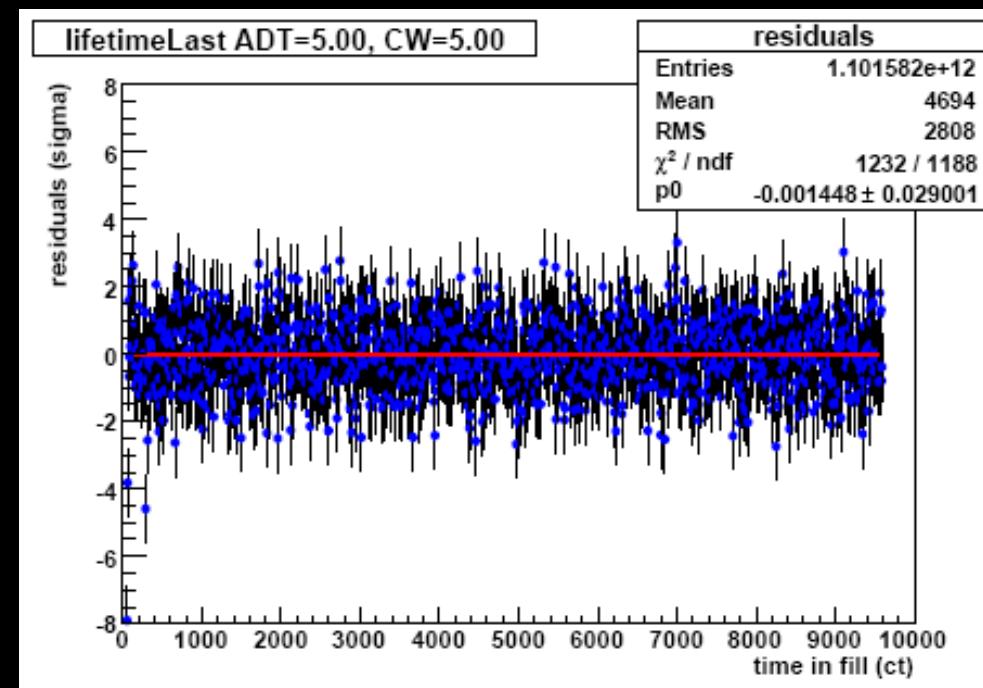
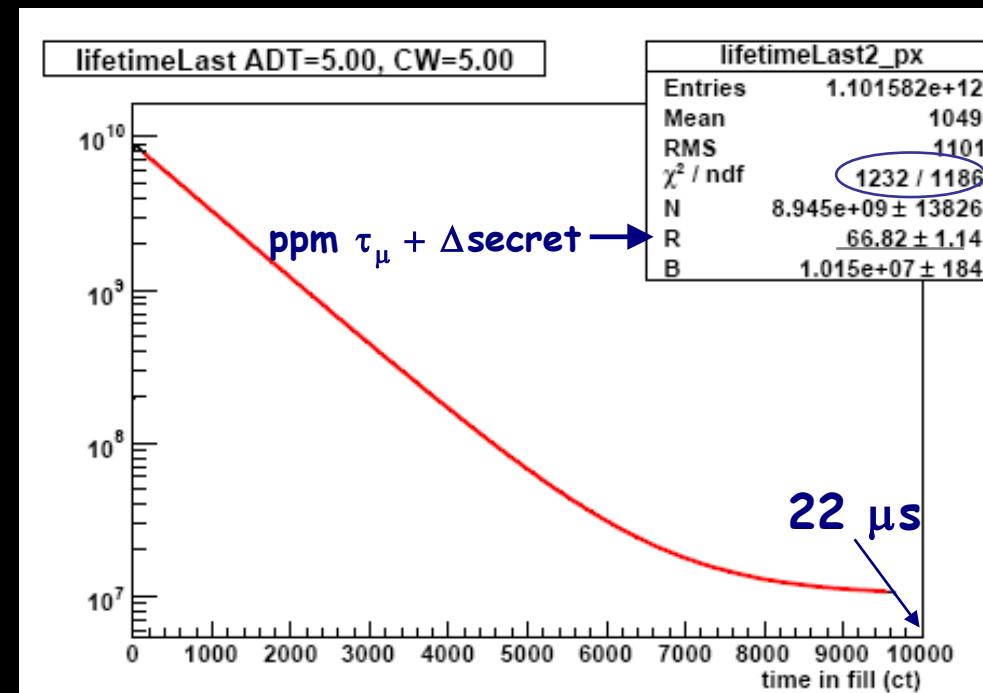
Incorporate  $\mu$ SR effects into the fit  
function. Fit each detector individually.

## AK-3, Sum of all detectors, sum of all 30000 runs

fit function

$$f(t) = N_0 e^{-t/\tau_\mu} + B$$

$$\tau_\mu = \tau_{\text{secret}}(1 + R/10^6)$$



## Quartz target, fit individual detectors

fit function

(most general form)

$$f(t) = N_0 \left[ 1 + A \vec{\mathcal{P}}(t) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

decay asymmetry

unit vector to detector

ensemble polarization

# Quartz target, fit individual detectors

fit function

(most general form)

$$f(t) = N_0 \left[ 1 + A \vec{\mathcal{P}}(t) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

decay asymmetry

unit vector to detector

ensemble polarization

resolve  $\mathcal{P}$  into two components relative to B-field

$$f(t) = N_0 \left[ 1 + A(\vec{\mathcal{P}}_1(t) + \vec{\mathcal{P}}_2(t)) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

transverse (precession+relaxation)

longitudinal (relaxation)

# Quartz target, fit individual detectors

fit function

(most general form)

$$f(t) = N_0 \left[ 1 + A \vec{\mathcal{P}}(t) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

decay asymmetry

unit vector to detector

ensemble polarization

resolve  $\mathcal{P}$  into two components relative to B-field

$$f(t) = N_0 \left[ 1 + A(\vec{\mathcal{P}}_1(t) + \vec{\mathcal{P}}_2(t)) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

transverse (precession+relaxation)

longitudinal (relaxation)

$$\mathcal{P}_1(t) = P_1 e^{-t/T_1} \quad P_1 \sim 0.0015, T_1 \sim 28\mu\text{s}$$

$$\mathcal{P}_2(t) = P_2 e^{-t/T_2} \cos(\omega t + \phi_0) \quad P_2 \sim 0.0025, T_2 \sim 4\mu\text{s}$$

# Quartz target, fit individual detectors

fit function

(most general form)

$$f(t) = N_0 \left[ 1 + A \vec{\mathcal{P}}(t) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

decay asymmetry

unit vector to detector

ensemble polarization

resolve  $\mathcal{P}$  into two components relative to B-field

$$f(t) = N_0 \left[ 1 + A(\vec{\mathcal{P}}_1(t) + \vec{\mathcal{P}}_2(t)) \hat{r}_D \right] e^{-t/\tau_\mu} + B$$

transverse (precession+relaxation)

longitudinal (relaxation)

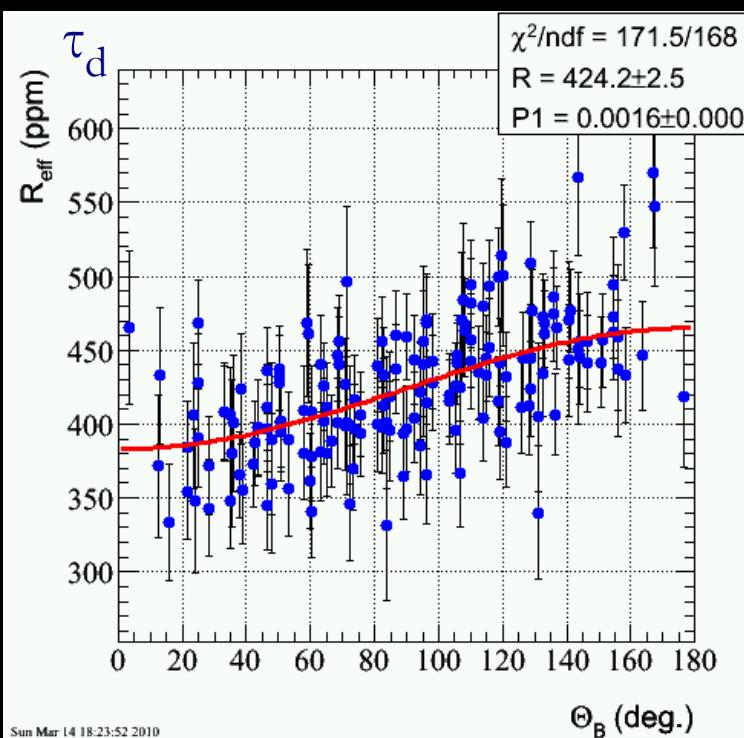
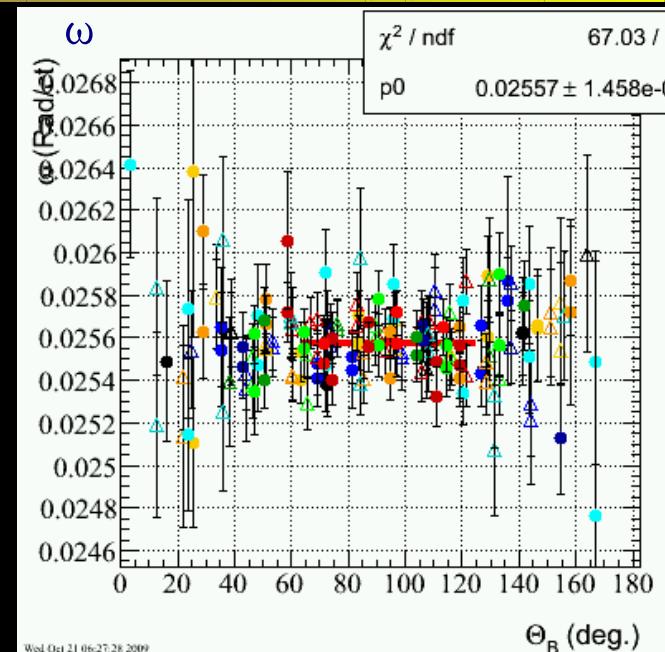
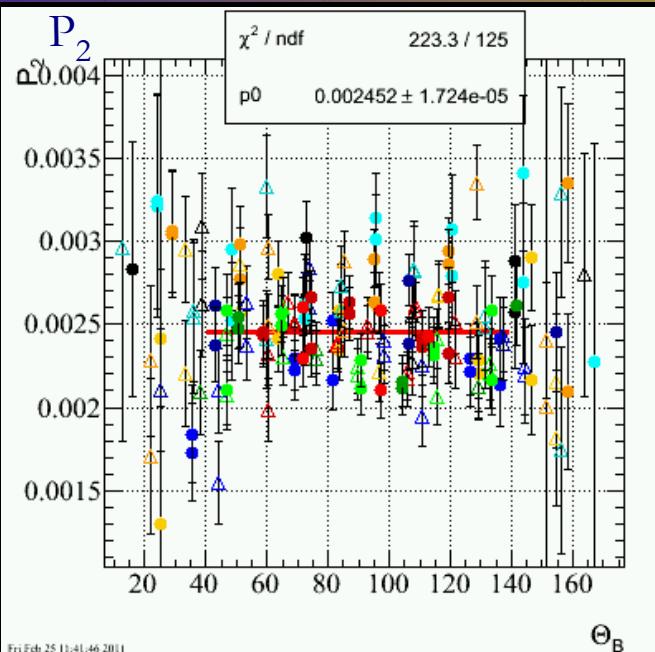
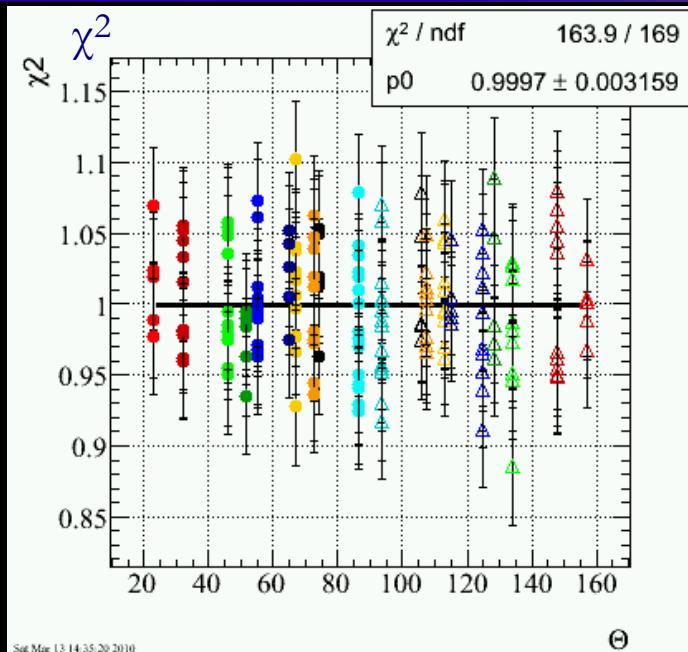
$\mathcal{P}_1(t) = P_1 e^{-t/T_1}$        $P_1 \sim 0.0015, T_1 \sim 28\mu\text{s}$   
 $\mathcal{P}_2(t) = P_2 e^{-t/T_2} \cos(\omega t + \phi_0)$        $P_2 \sim 0.0025, T_2 \sim 4\mu\text{s}$

practical realization fit for muon disappearance time, then fit for  $\tau_\mu$

$$f(t) = N_0 \left[ 1 + AP_2 e^{-t/T_2} \cos(\omega t + \phi_0) \right] e^{-t/\tau_d} + B \quad \rightarrow 170 \text{ values of } \tau_d$$

$$\tau_d = \tau_\mu \left( 1 - A \frac{\tau_\mu}{T_1} \vec{P}_1 \hat{r}_D \right)$$

# fit results vs. detector position relative to B-field

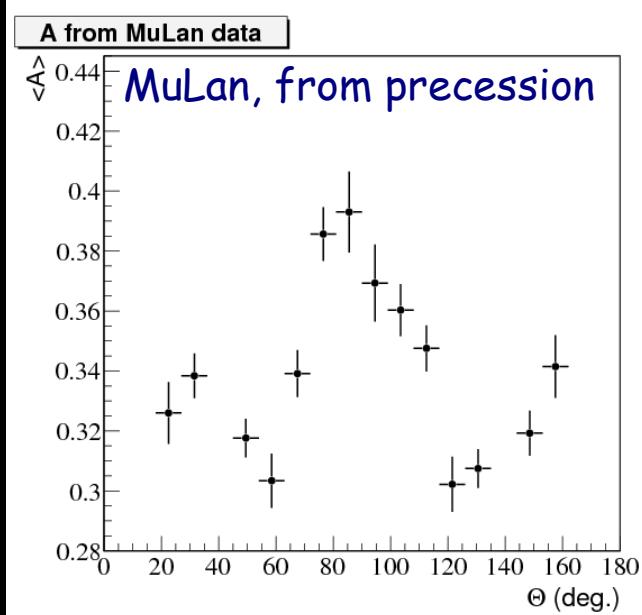
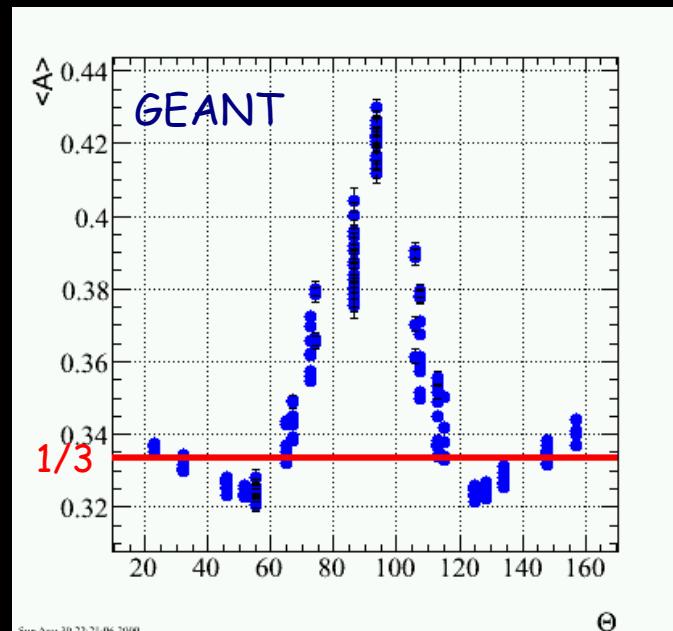
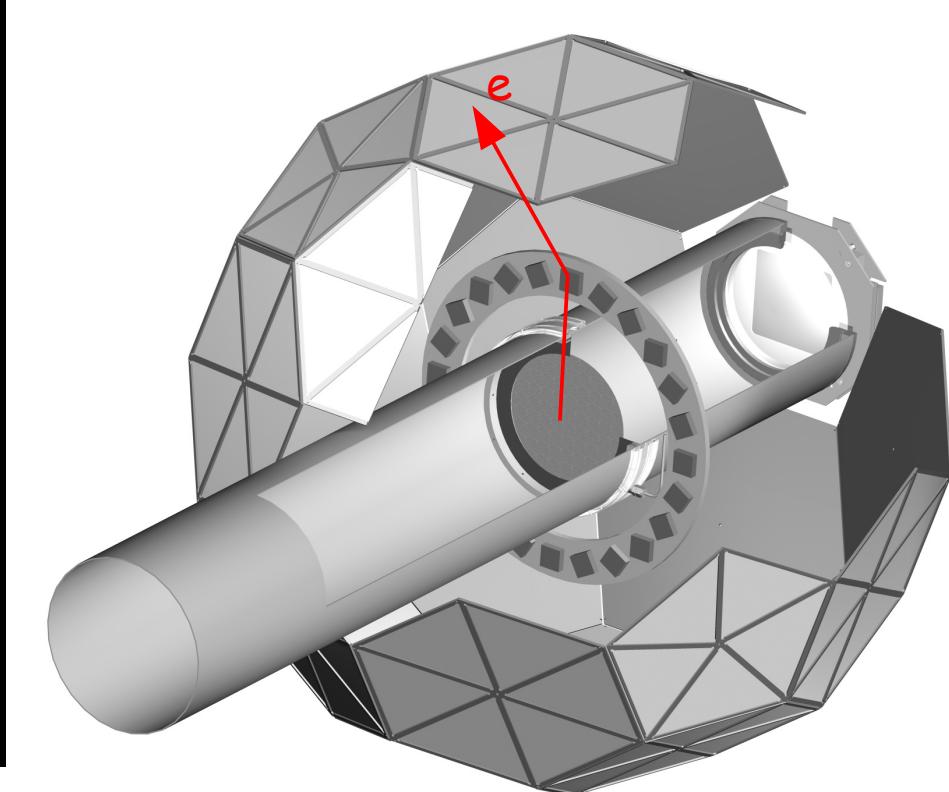
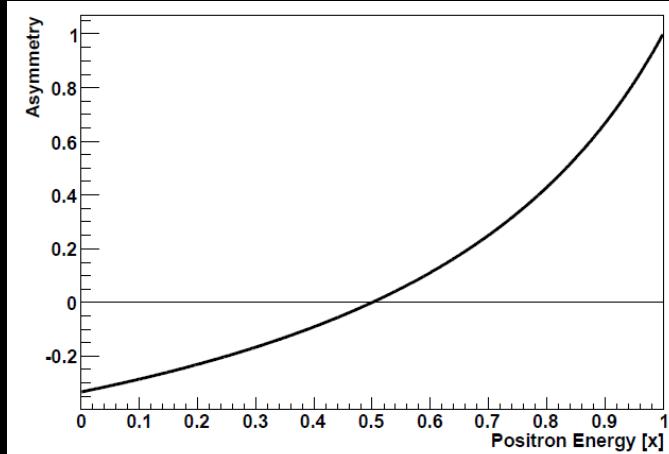


$$\tau_d = \tau_\mu \left( 1 - A \frac{\tau_\mu}{T_1} \vec{P}_1 \hat{r}_D \right)$$

# A small complication: decay asymmetry

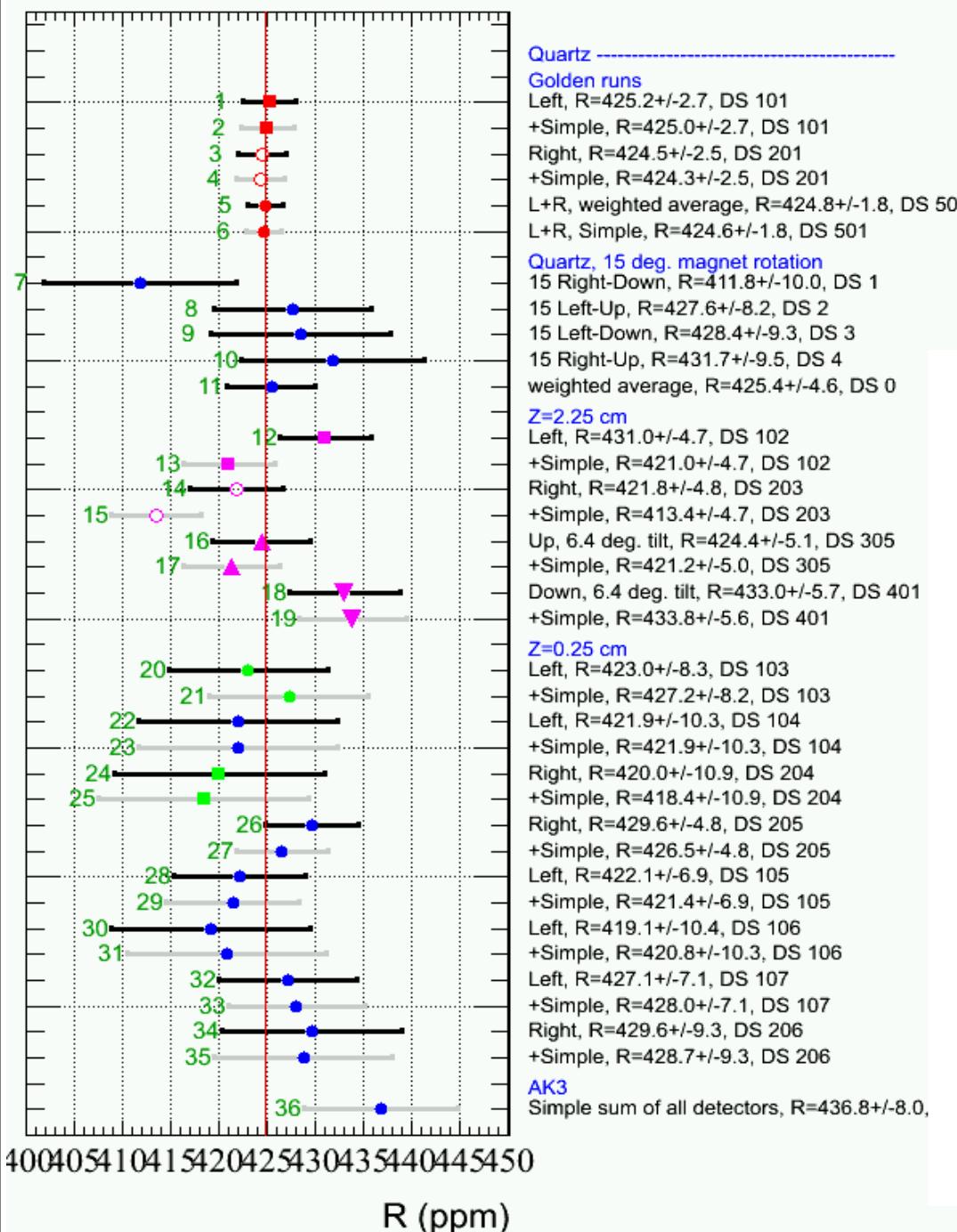
$$f(t) = N_0 \left[ 1 + AP_2 e^{-t/T_2} \cos(\omega t + \phi_0) \right] e^{-t/\tau_d} + B$$

Energy-integrated  $A = 1/3$

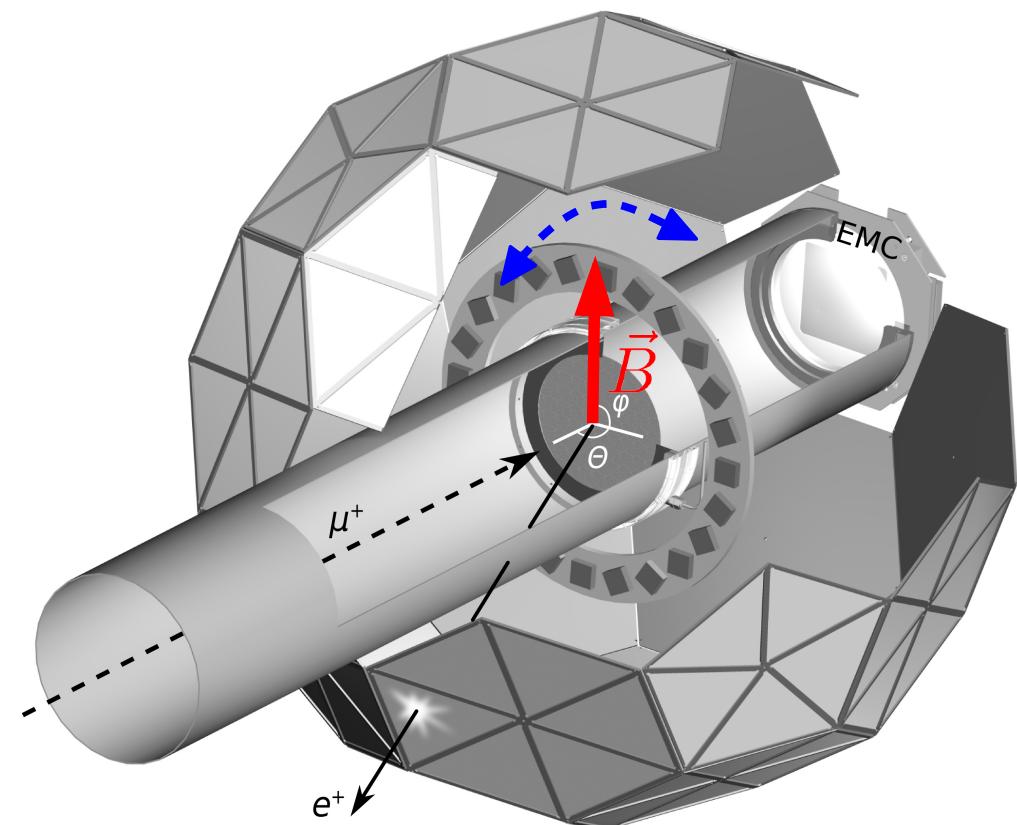


Plugging in GEANT  
asymmetries instead of a  
common  $A=1/3$   
asymmetry changes muon  
lifetime by 0.1 ppm

# Consistency checks

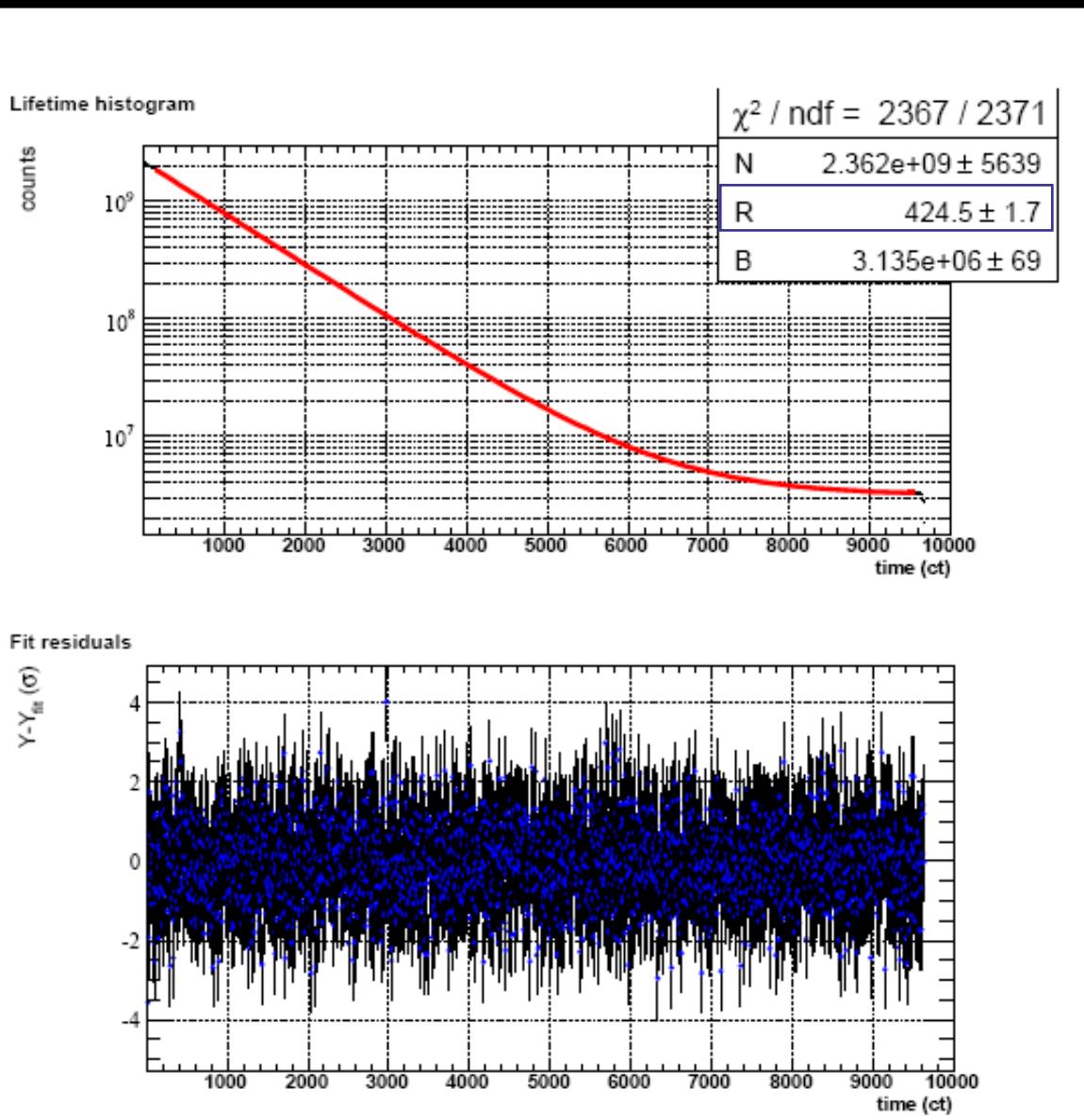


Consistency against MANY special runs, where we varied target, magnet, ball

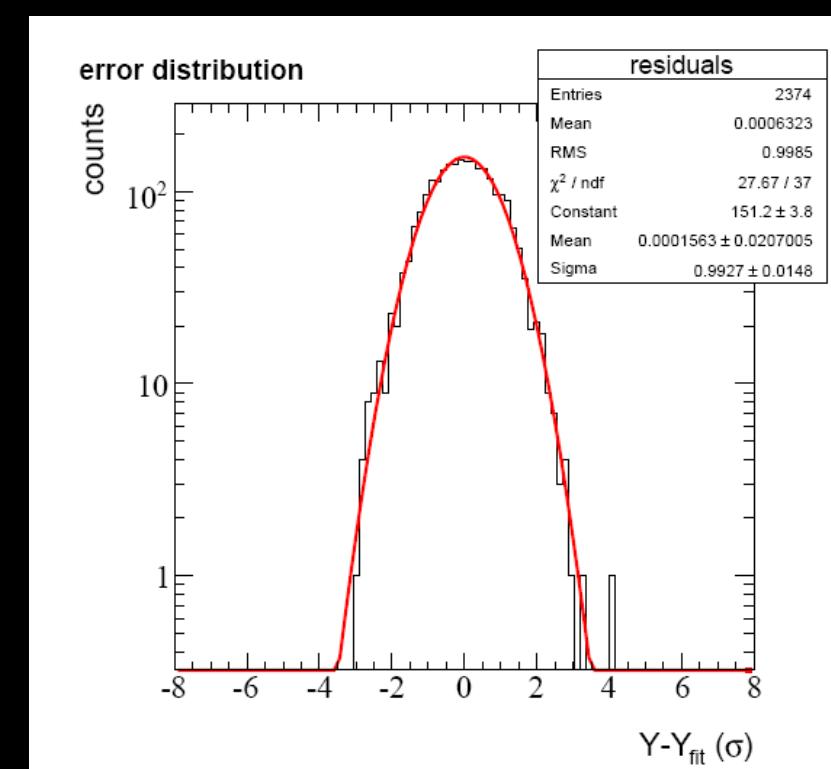


# Quartz target: fit the sum...

Quartz data fits well as a simple sum, exploiting the symmetry of the detector.



The deviation from the "fit each detector method" is **0.3ppm**



$$f(t) = N_0 \left[ 1 + AP_2 e^{-t/T_2} \cos(\omega t + \phi_0) \right] e^{-t/\tau_d} + B$$

$$\tau_d = \tau_\mu \left( 1 - A \frac{\tau_\mu}{T_1} \vec{P}_1 \hat{r}_D \right)$$

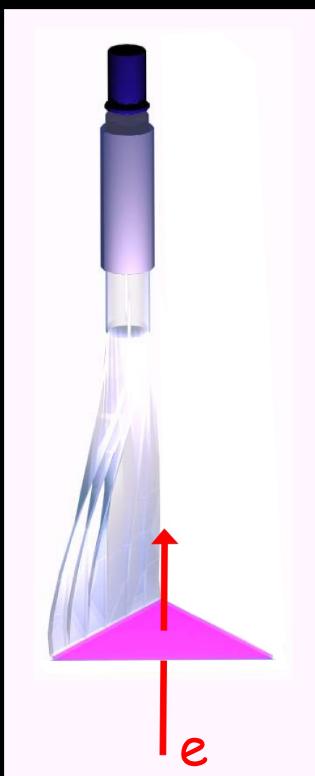
parameter	$\delta\tau_\mu$ (ppm)
$\omega$	0.003
$T_2$	0.001
$\hat{P}_1$ <sup>1)</sup>	0.004
beam $x$ <sup>2)</sup>	0.18
beam $y$ <sup>2)</sup>	<0.001
beam $z$ <sup>2)</sup>	<0.001
$A$ <sup>3)</sup>	0.05
total error	0.20

<sup>1)</sup> for  $\pm 2^\circ$  variation of the direction of vector  $\hat{P}_1$

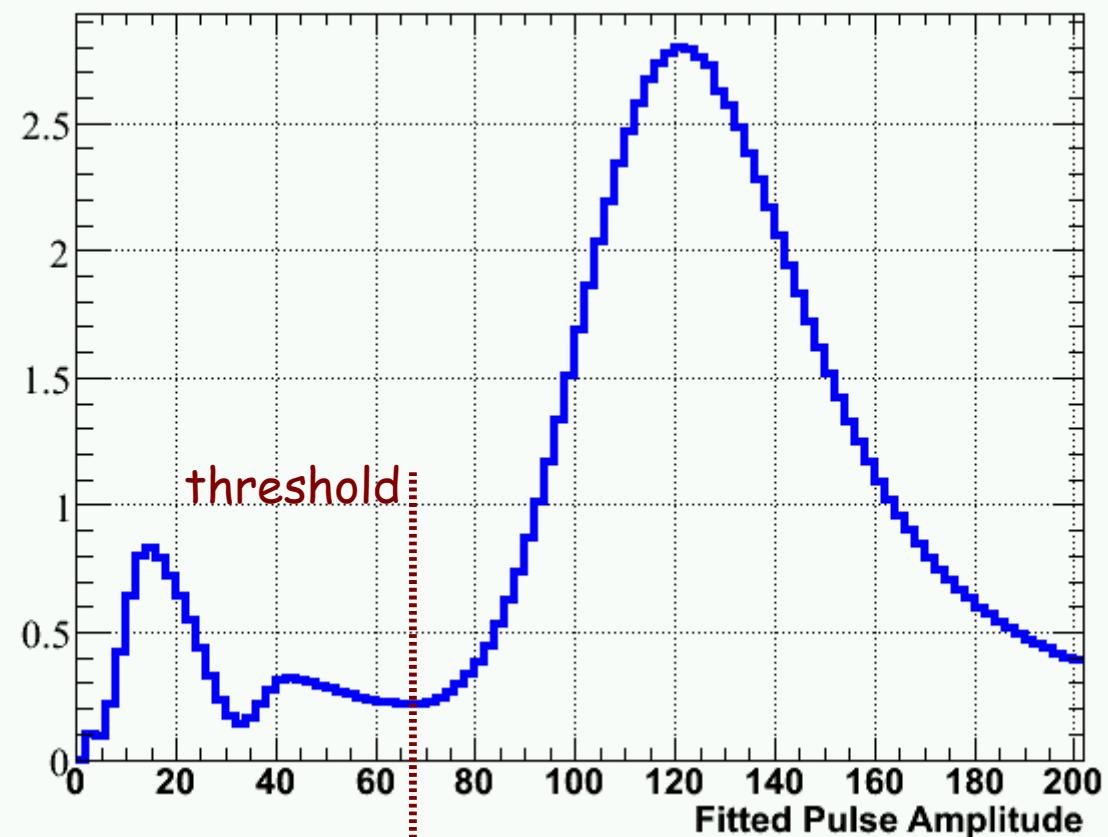
<sup>2)</sup> for  $\pm 2$  mm variation of the position of the beam

<sup>3)</sup> change between GEANT4 or MuLan asymmetries

# Gain stability

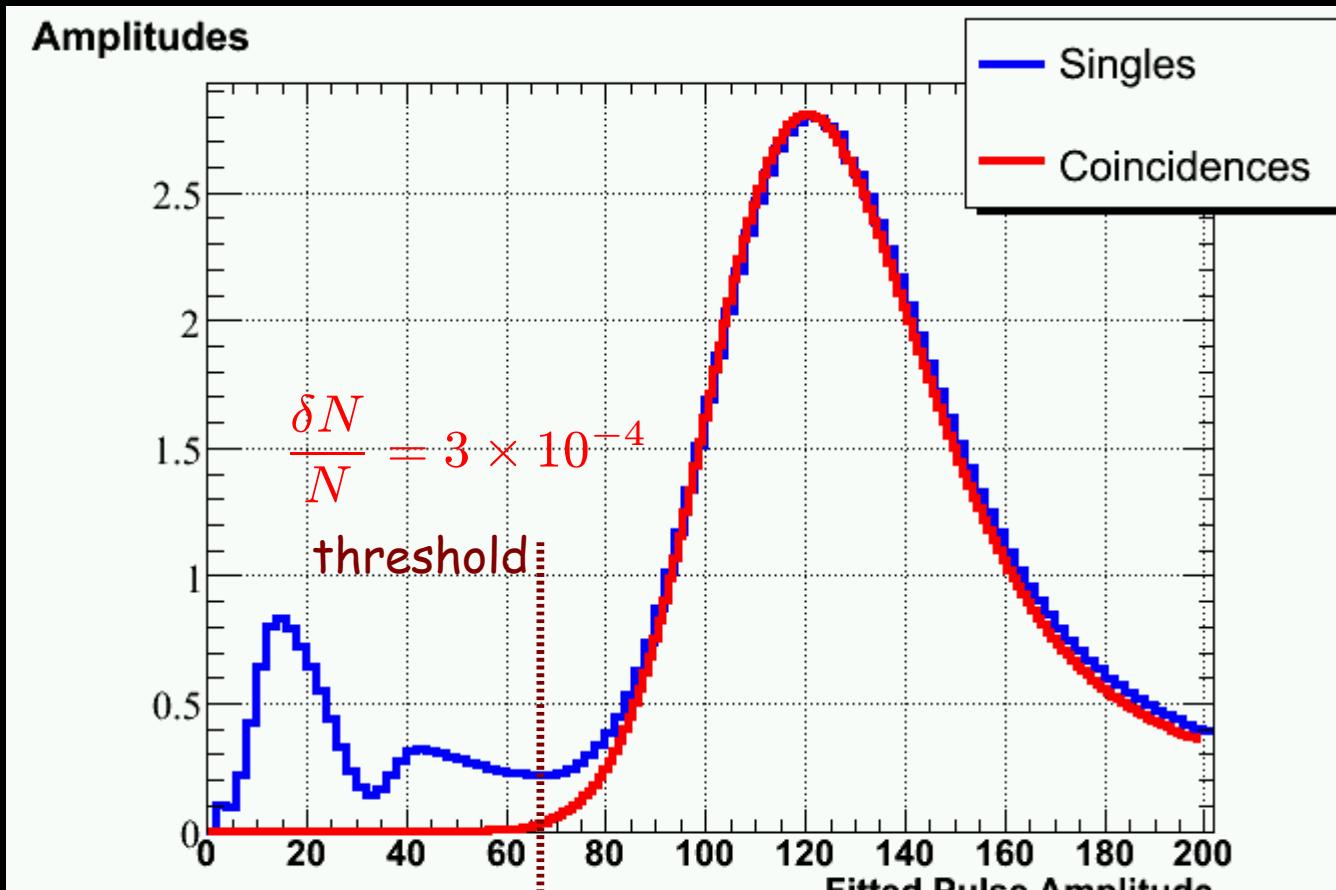


Raw singles

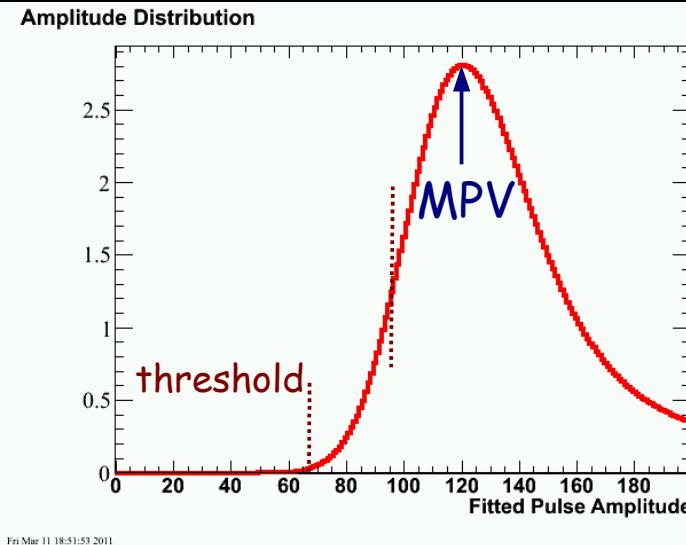


Mon Apr 5 18:38:19 2010

## coincidences suppress background



# Correction for early-to-late gain variation



Gain correction is 0.5 ppm  
shift with 0.25 ppm  
uncertainty.

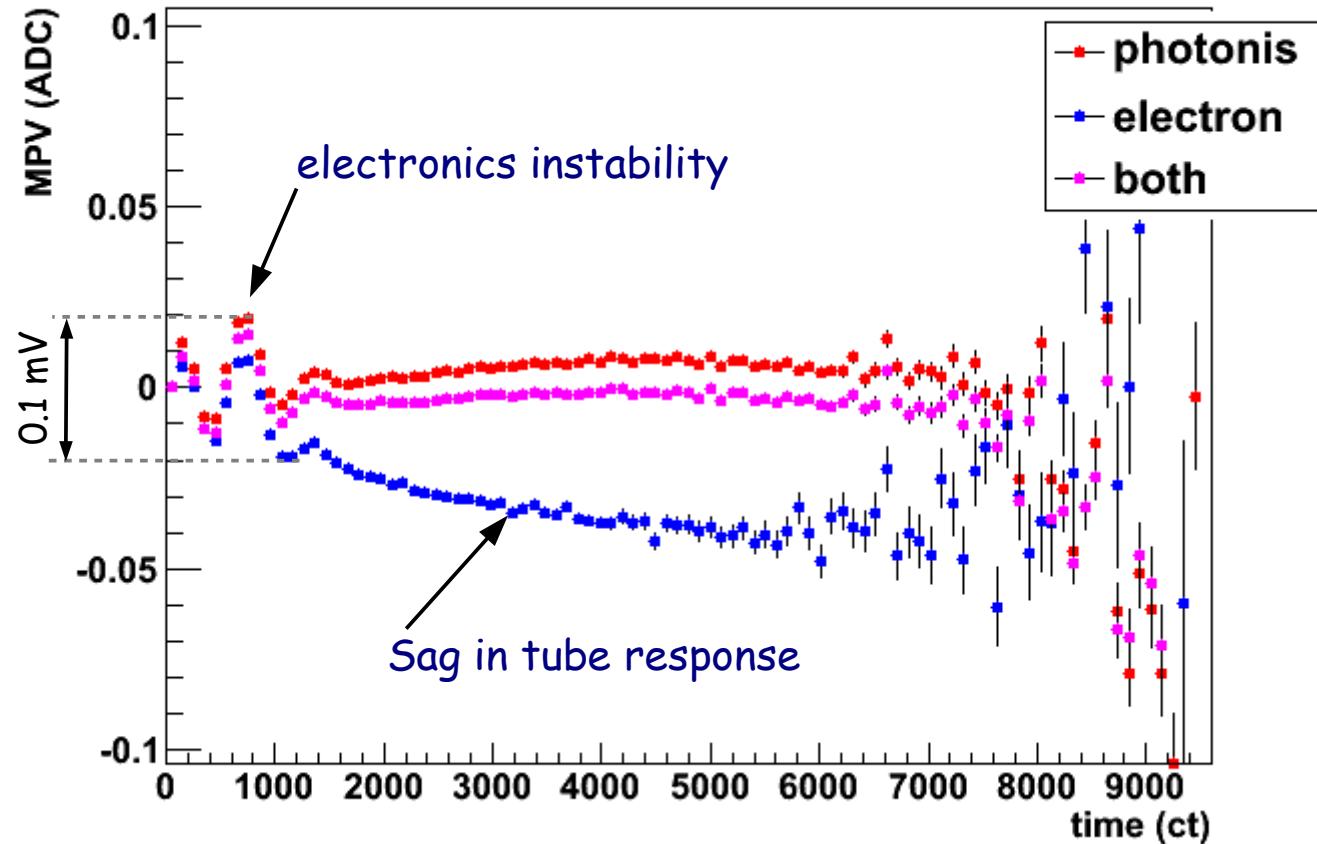
## Correction

- 1) Gain vs. time variation is derived from the stability of the peak of the fit to pulse amplitude distribution
- 2) Extrapolate from MPV to threshold

## Consistency check

Raise the threshold to amplify the effect

Gain is photomultiplier tube type dependent



# Timing stability

# Master clock

The clock was provided by an Agilent E4400B RF Signal Generator, which was stable during the run and found to be accurate to 0.025 ppm.

Agilent E4400 RF Signal Generator



$f = 450.87649126$  MHz

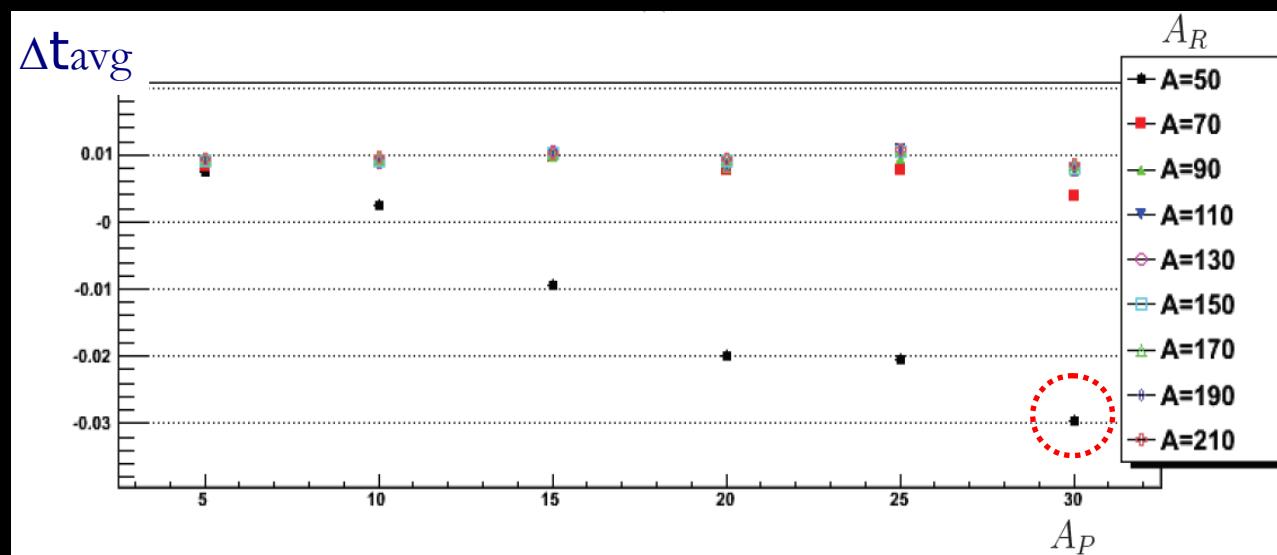
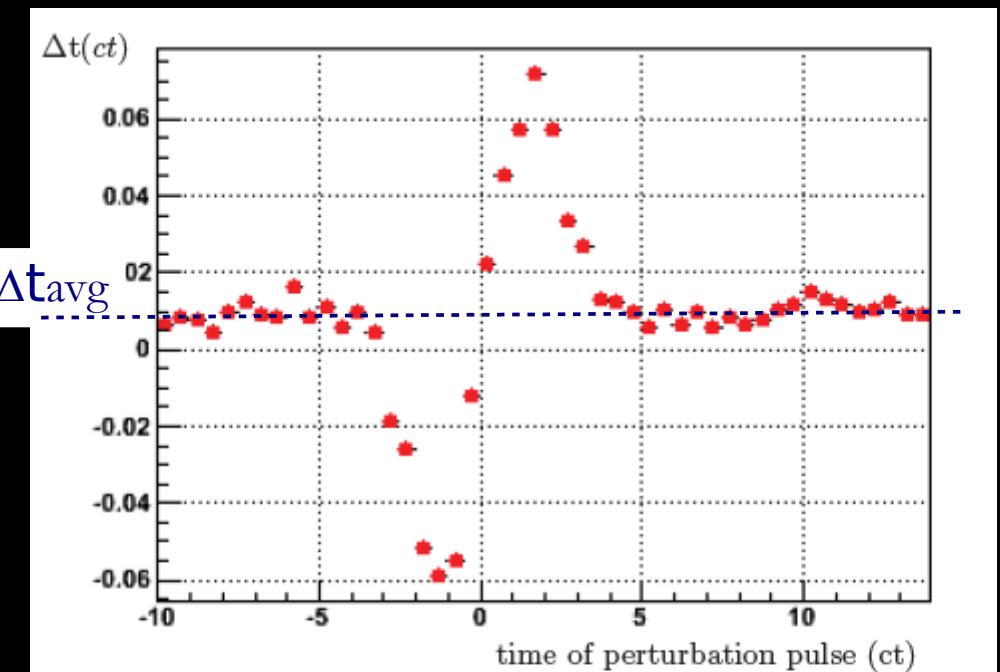
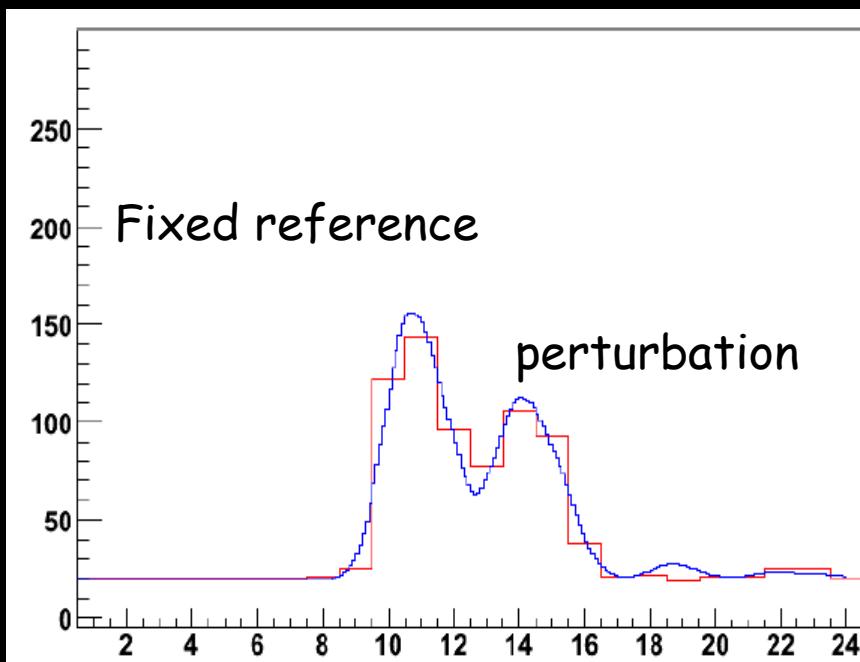
- Checked for consistency throughout the run.
- Compared to Quartzlock A10-R rubidium frequency standard.
- Compared to calibrated frequency counter
- Different blinded frequencies in 2006 and 2007

Comparison	10 MHz	60 MHz
Frequency counter	$1 \times 10^{-8}$	$2 \times 10^{-8}$
Rubidium atomic clock	$4 \times 10^{-8}$	$3 \times 10^{-8}$

Average difference = 0.025 ppm

# Nearby unseen pulses perturb the time of main pulses

## software stability studied with simulations

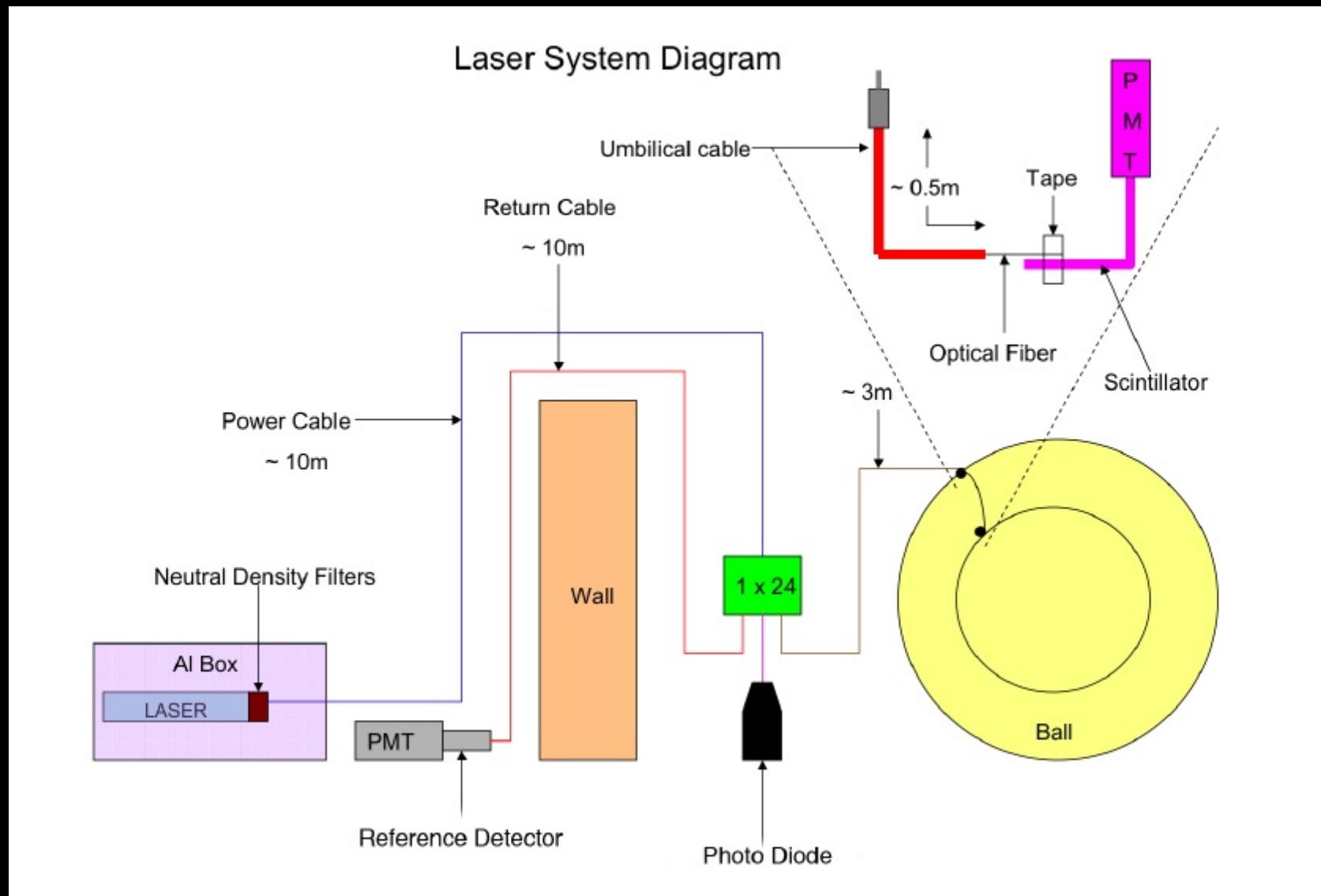


Estimated pull:

$$\frac{(0.03 \text{ ct}) \times (0.25\% \text{ pileup})}{(1000 \text{ ct} / \tau_\mu)} = 0.075 \text{ ppm}$$

D. M. Webber

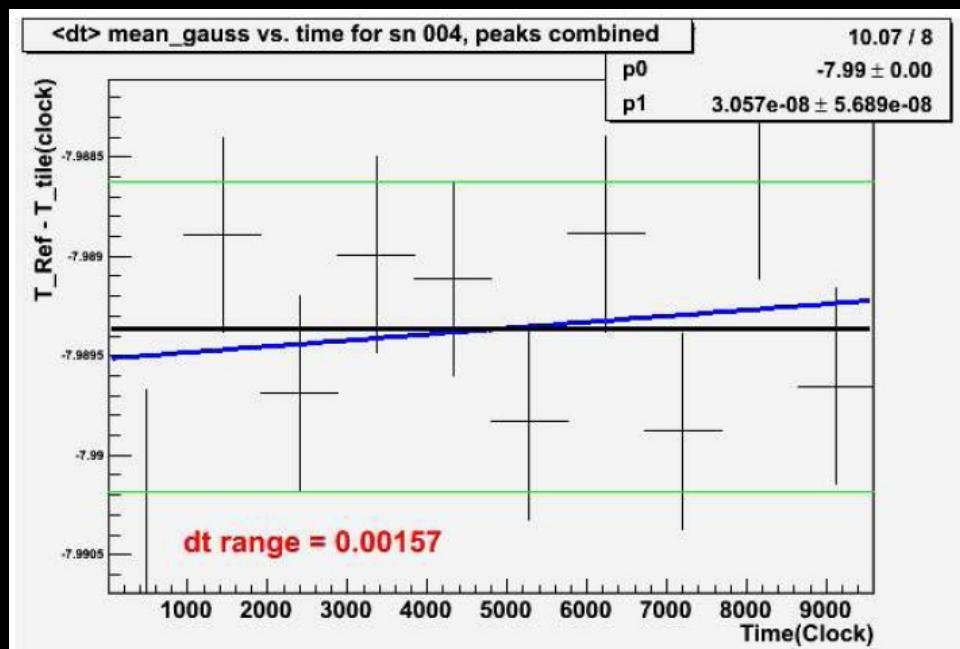
# Hardware stability studied with Laser pulses



Laser: LN203C sealed nitrogen/dye laser made by Laser Photonics Inc.

Peak power 167kW, spectral output 337.1nm; pulse width: 600 ps (FWHM)

# Timing stability from laser data

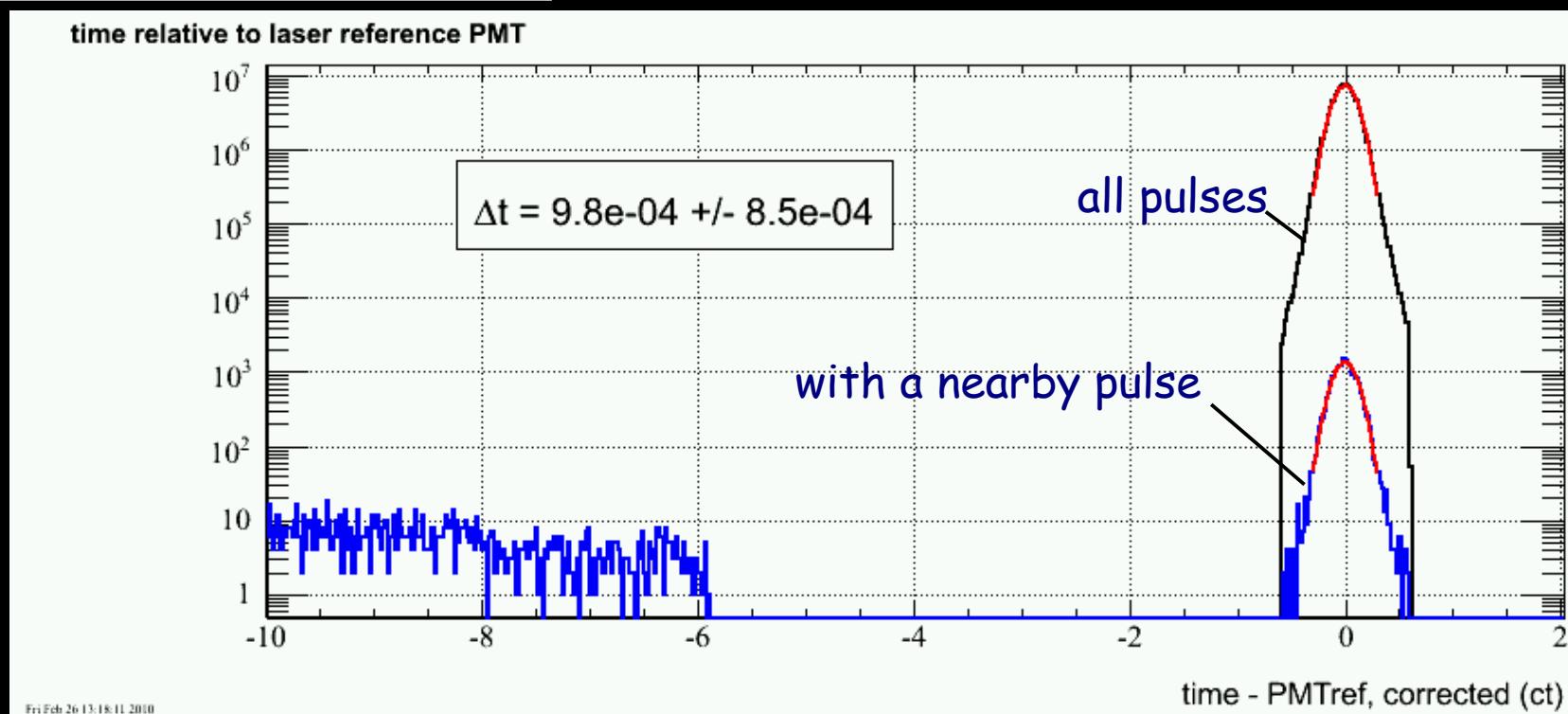


early-to-late timing shifts

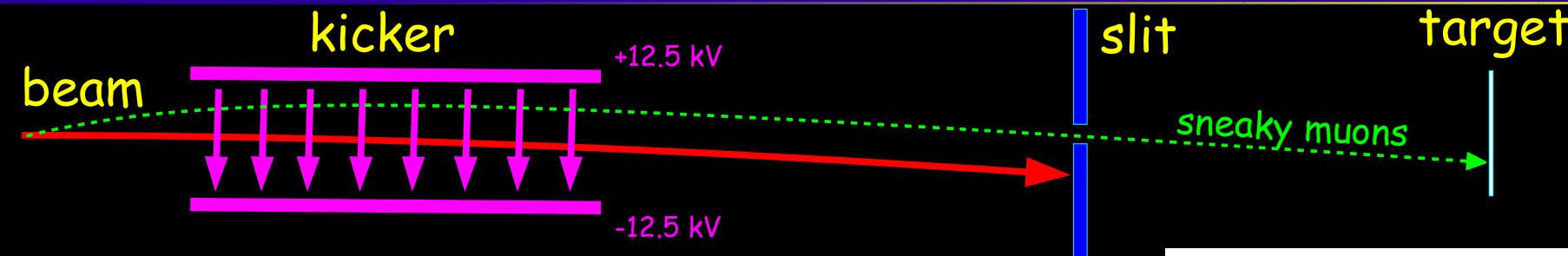
0.1 ppm, limited by statistics

timing shifts due to nearby pulses

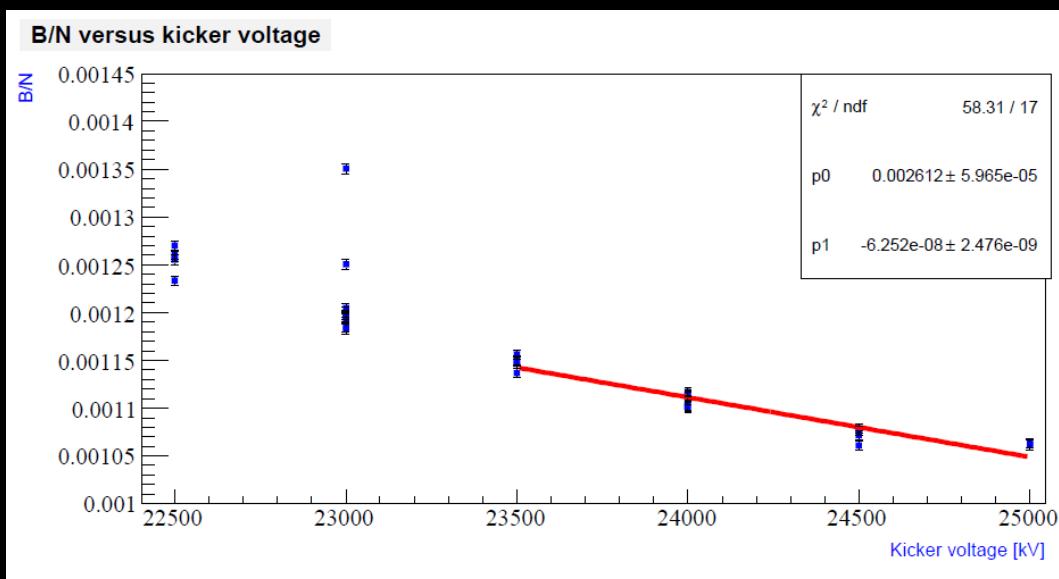
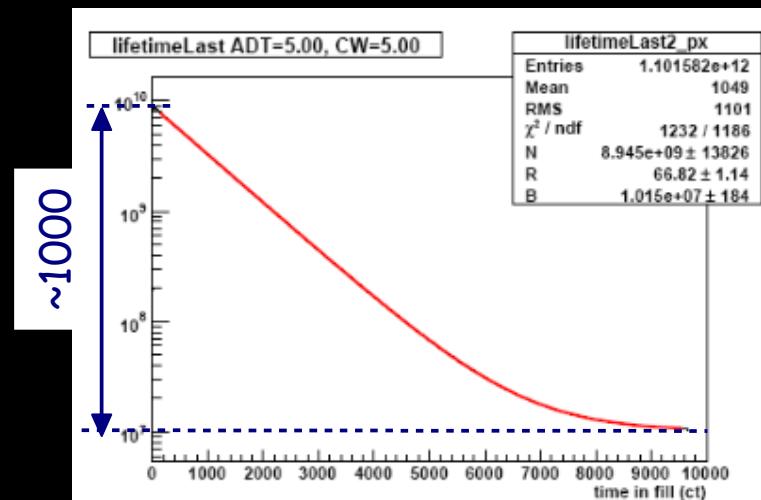
tiny



# Background stability



any variation of the plate high voltage during the measurement period leads to a deviation of a flat accidental background.



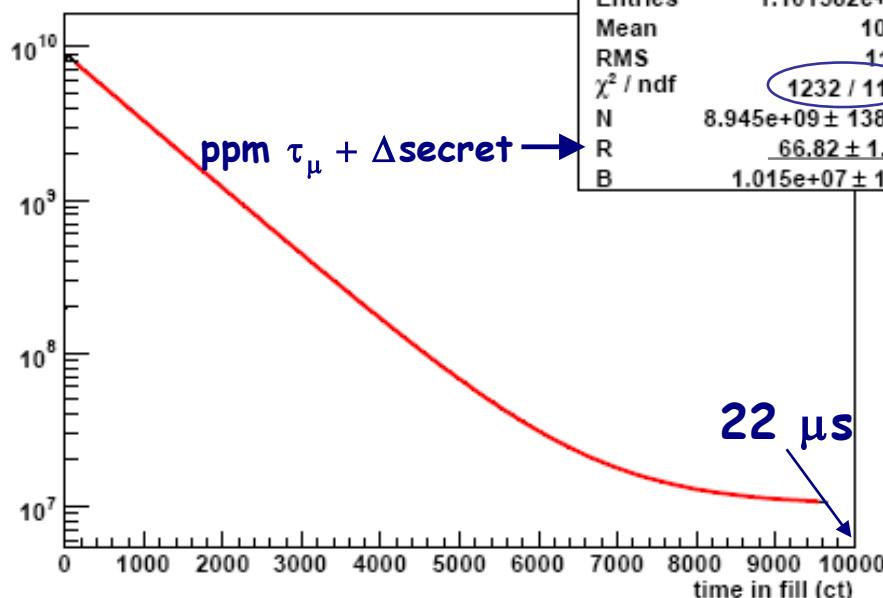
Systematic error stemming from a potential voltage drift on the kicker deflector plates

2006: 0.2 ppm

2007: 0.07 ppm

# Consistency checks: fit quality

lifetimeLast ADT=5.00, CW=5.00



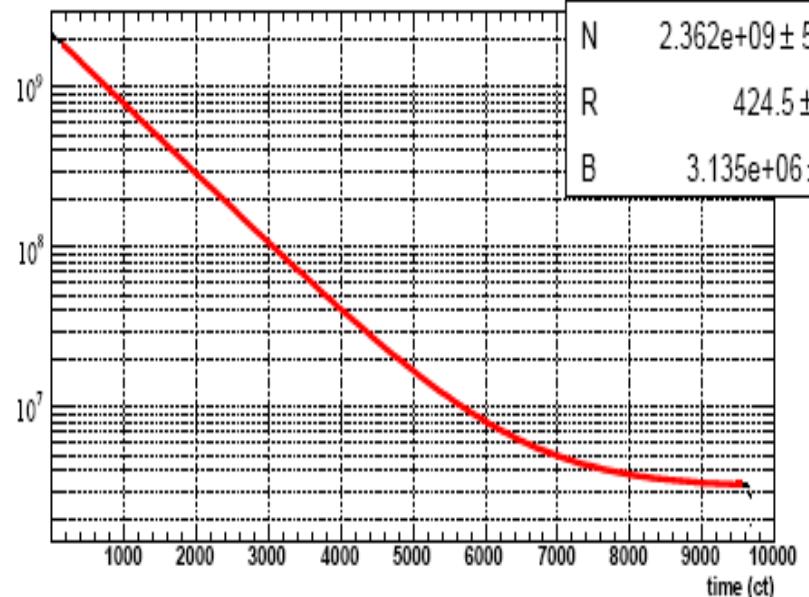
Lifetime histogram

$\chi^2 / \text{ndf} = 2367 / 2371$

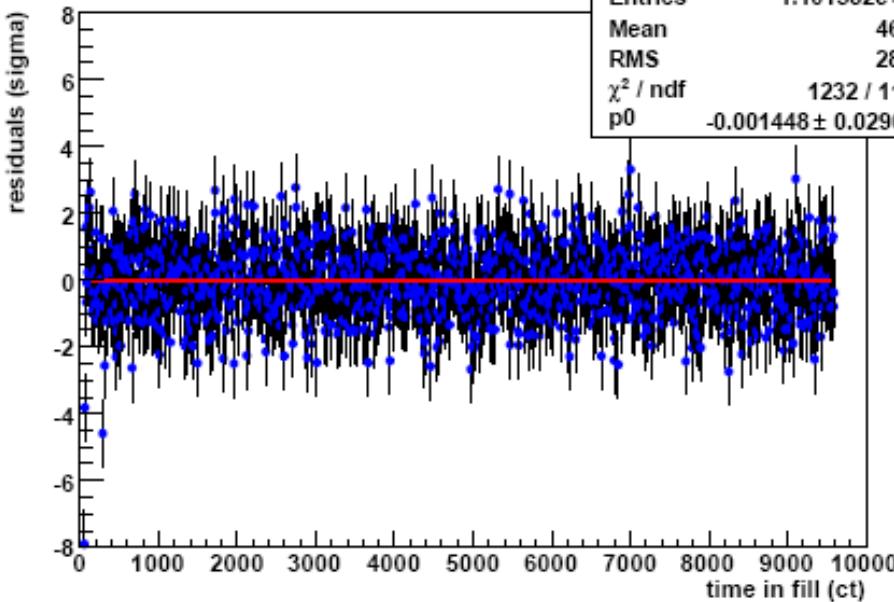
N  $2.362\text{e+09} \pm 5639$

R  $424.5 \pm 1.7$

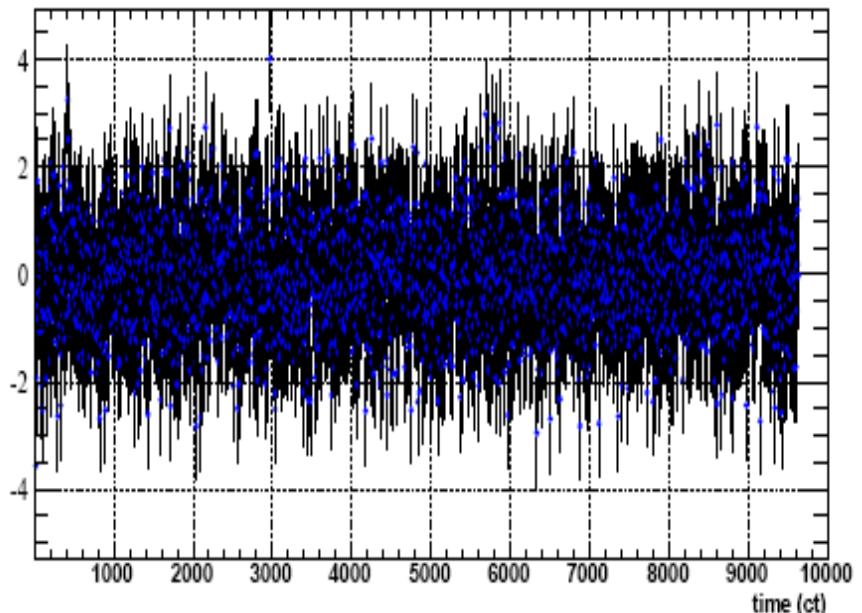
B  $3.135\text{e+06} \pm 69$



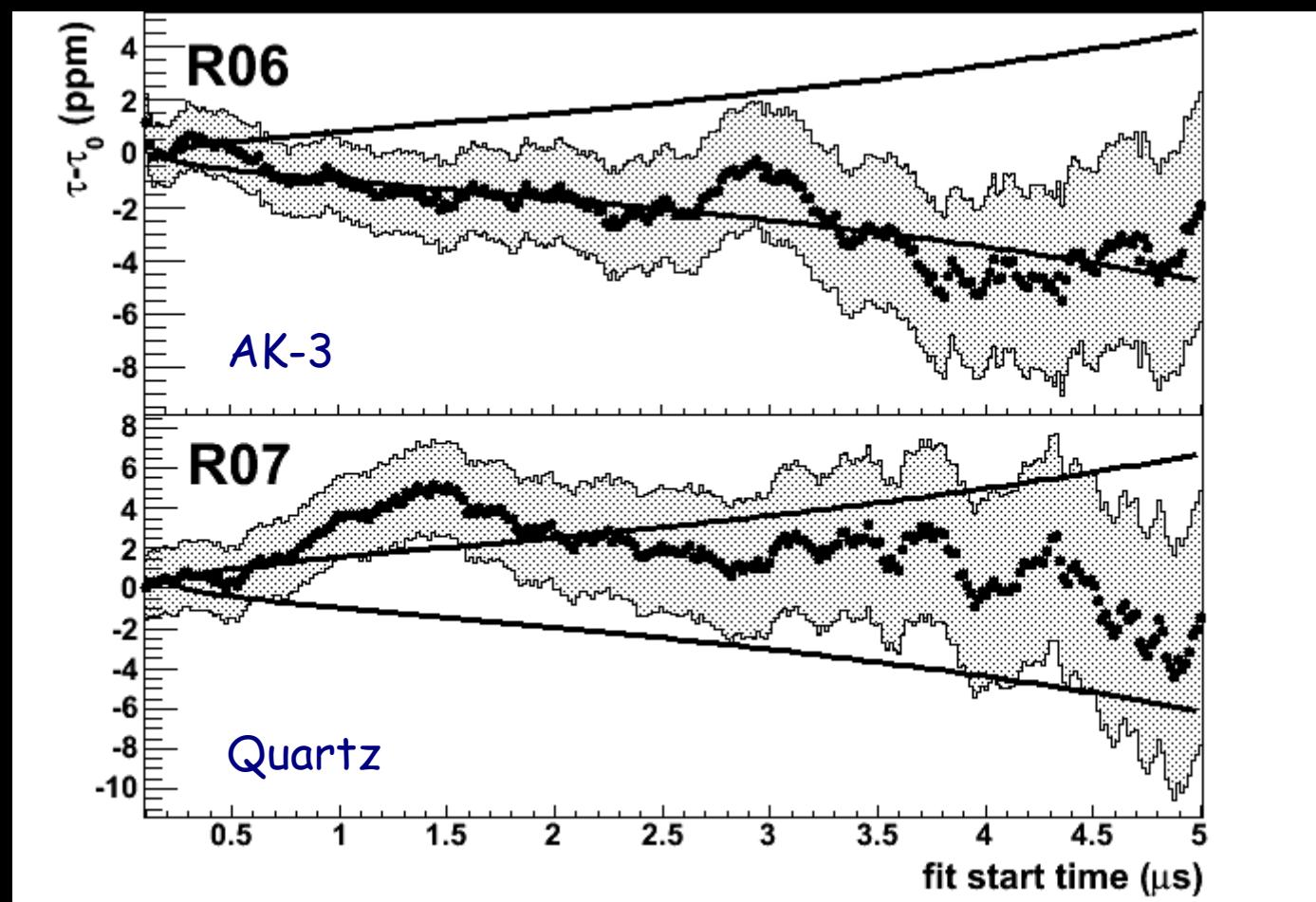
lifetimeLast ADT=5.00, CW=5.00



Fit residuals



## Consistency checks: Fit start time scan



# Final Errors and Numbers

ppm units

Effect	2006	2007	Comment
Kicker extinction stability	0.20	0.07	Voltage measurements of plates
Residual polarization	0.10	0.20	Long relax; quartz spin cancelation
Upstream muon stops		0.10	Upper limit from measurements
Overall gain stability:		0.25	MPV vs time in fill; includes:
Short time; after a pulse			MPVs in next fill & laser studies
Long time; during full fill			Different by PMT type
Electronic ped fluctuation			Bench-test supported
Unseen small pulses			Uncorrected pileup effect → gain
Timing stability		0.12	Laser with external reference ctr.
Pileup correction		0.20	Extrapolation to zero ADT
Clock stability		0.03	Calibration and measurement
Total Systematic	0.42	0.42	Highly correlated for 2006/2007
Total Statistical	1.14	1.68	

$$\tau(R06) = 2\ 196\ 979.9 \pm 2.5 \pm 0.9 \text{ ps}$$

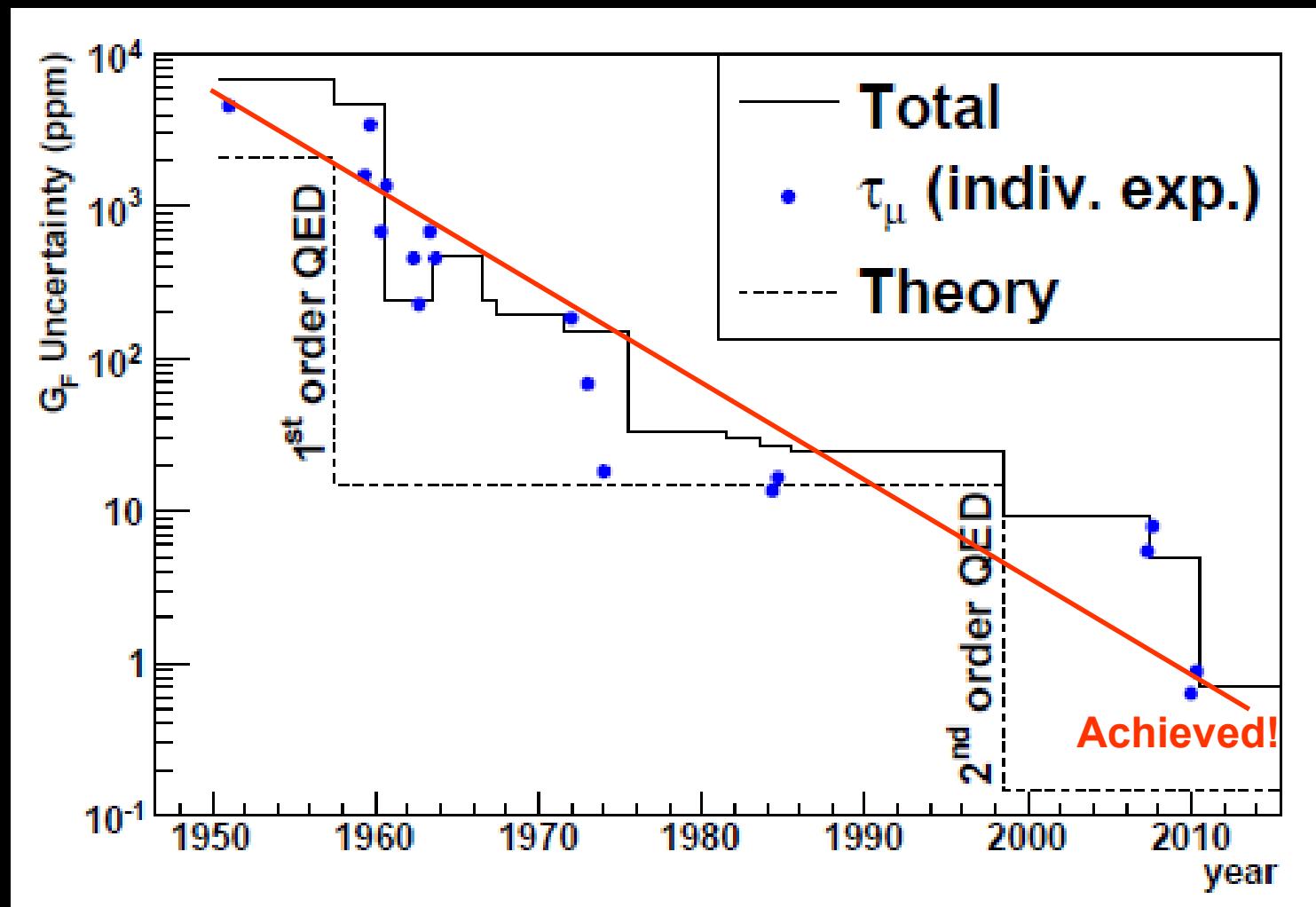
$$\tau(R07) = 2\ 196\ 981.2 \pm 3.7 \pm 0.9 \text{ ps}$$

$$\tau(\text{Combined}) = 2\ 196\ 980.3 \pm 2.2 \text{ ps (1.0 ppm)}$$

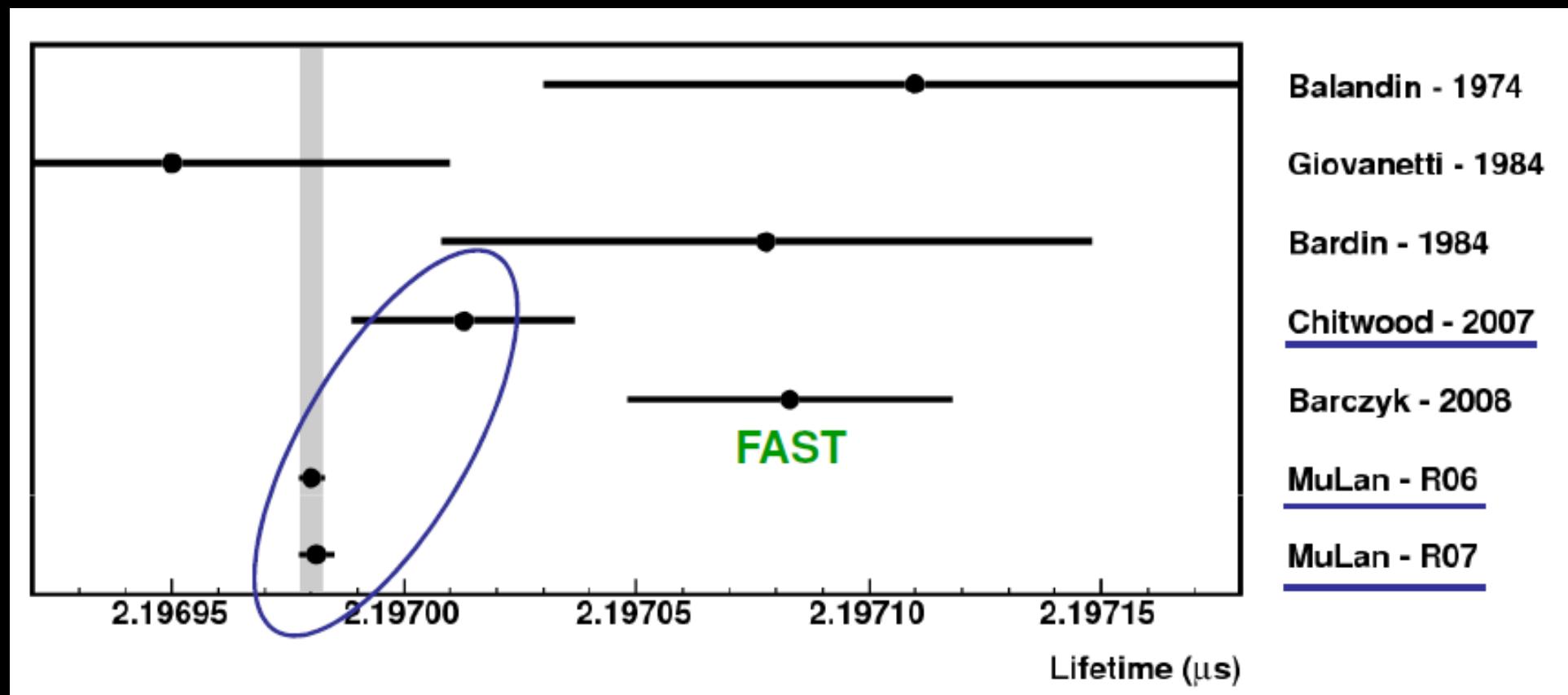
$$\Delta\tau(R07 - R06) = 1.3 \text{ ps}$$

New  $G_F$

$$G_F(\text{MuLan}) = 1.166\ 378\ 8(7) \times 10^{-5} \text{ GeV}^{-2} \text{ (0.6 ppm)}$$



## Lifetime "history"



# MuLan Collaborators



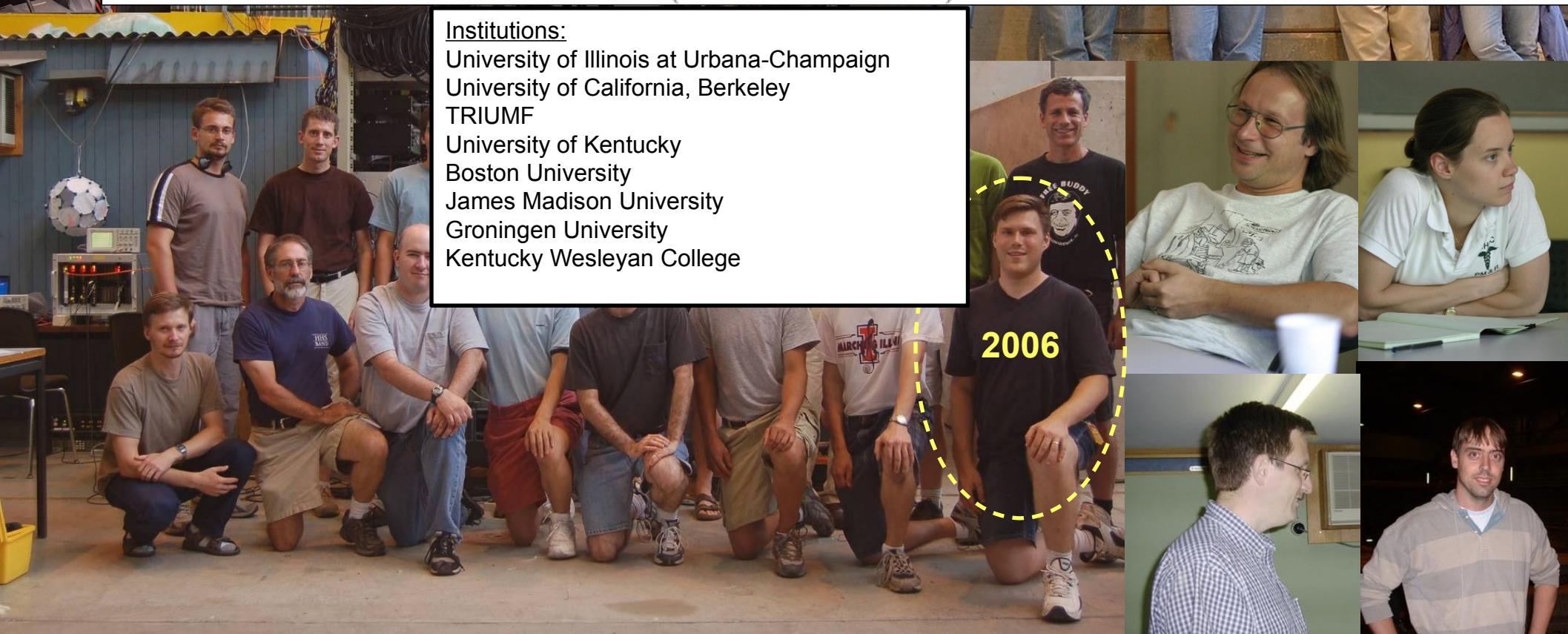
# MuLan Collaborators



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(MuLan Collaboration)

## Institutions:

University of Illinois at Urbana-Champaign  
University of California, Berkeley  
TRIUMF  
University of Kentucky  
Boston University  
James Madison University  
Groningen University  
Kentucky Wesleyan College



# Thank you!

